Geodesics on Hyperbolic Once-Punctured Tori

FYP Introductory Talk

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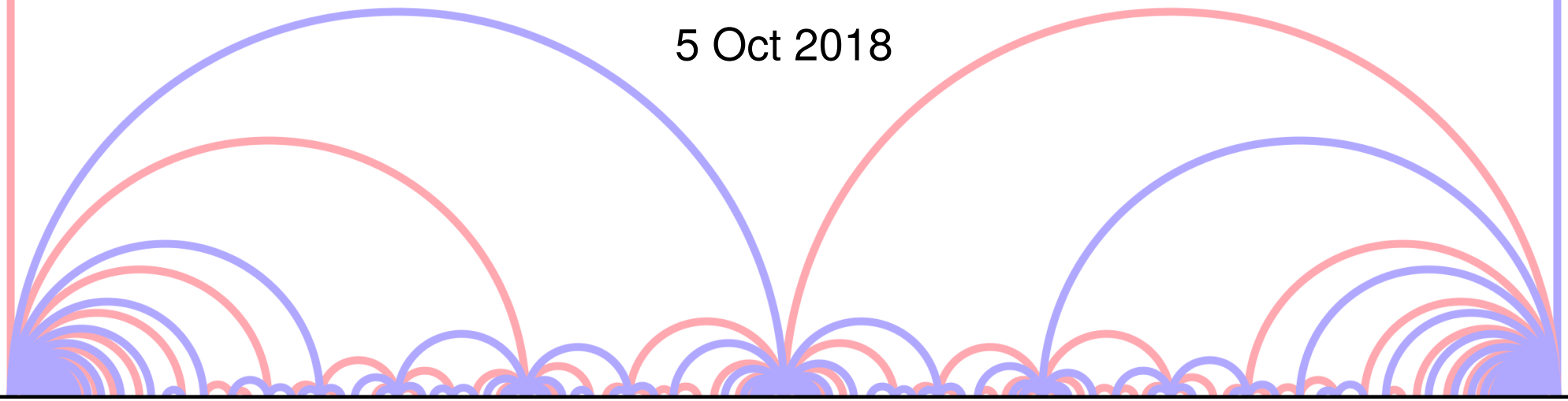


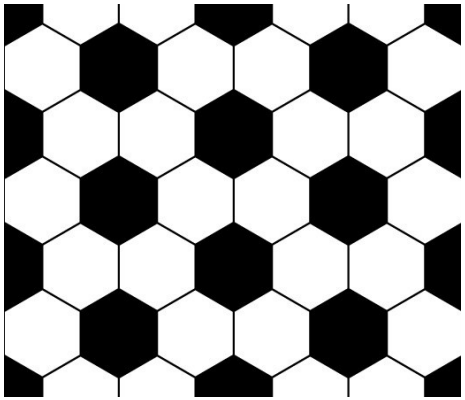
Geometry

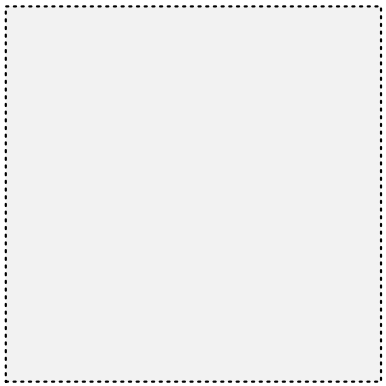
- Surfaces with notion of length and angle
  (= Riemannian metric)
- Measure deviation from Euclidean plane
  eg. Area excess/deficit
  (= Gaussian curvature)
- Which surfaces have constant curvature?
  What can we say about them?

Surfaces of constant curvature

$K \equiv 0$

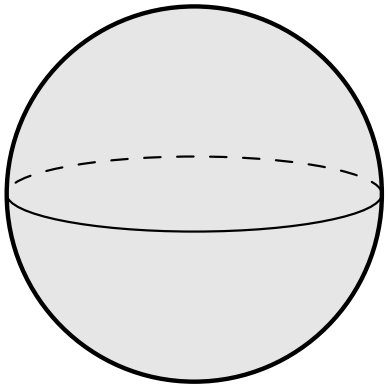
Euclidean plane





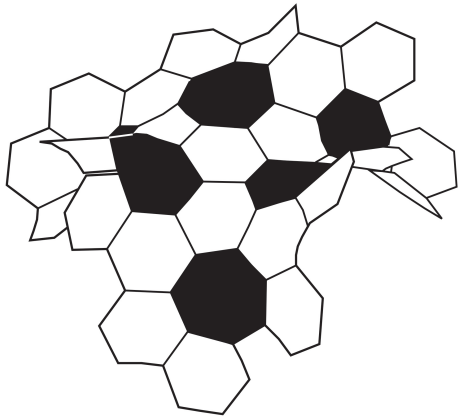
$K \equiv 1$

Sphere

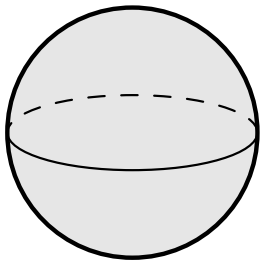


$K \equiv -1$

Hyperbolic plane



Drawing the hyperbolic plane



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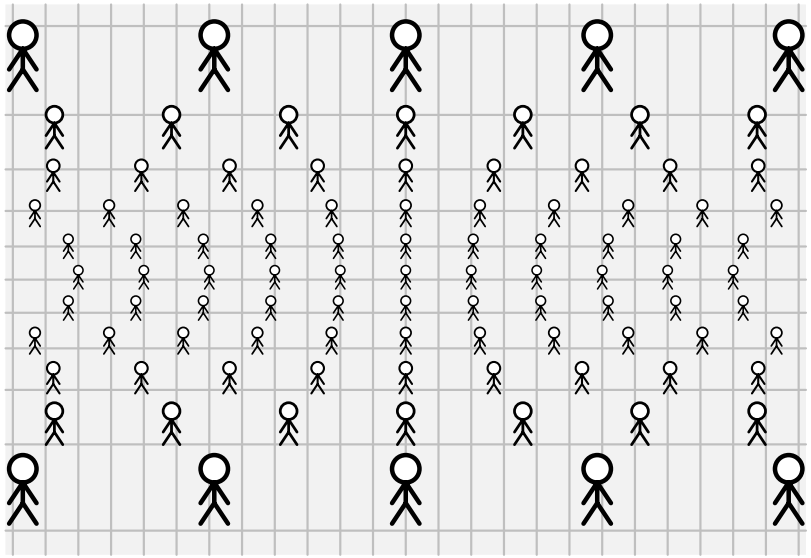

Mercator projection

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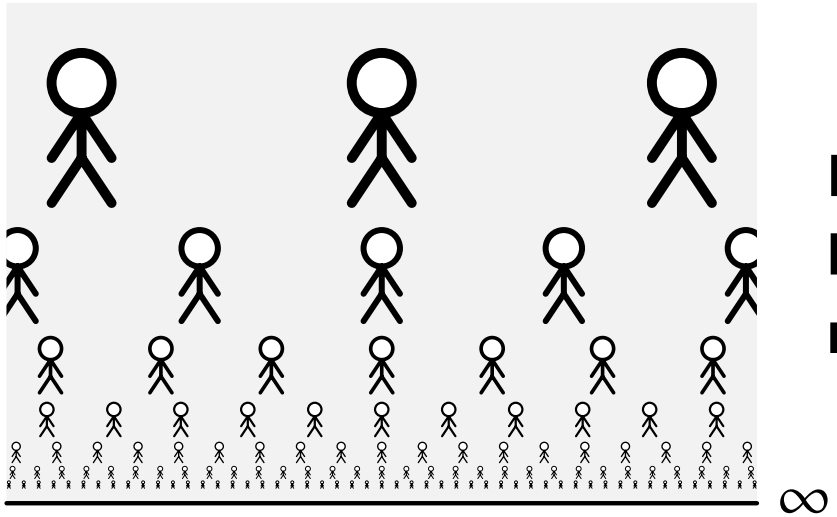


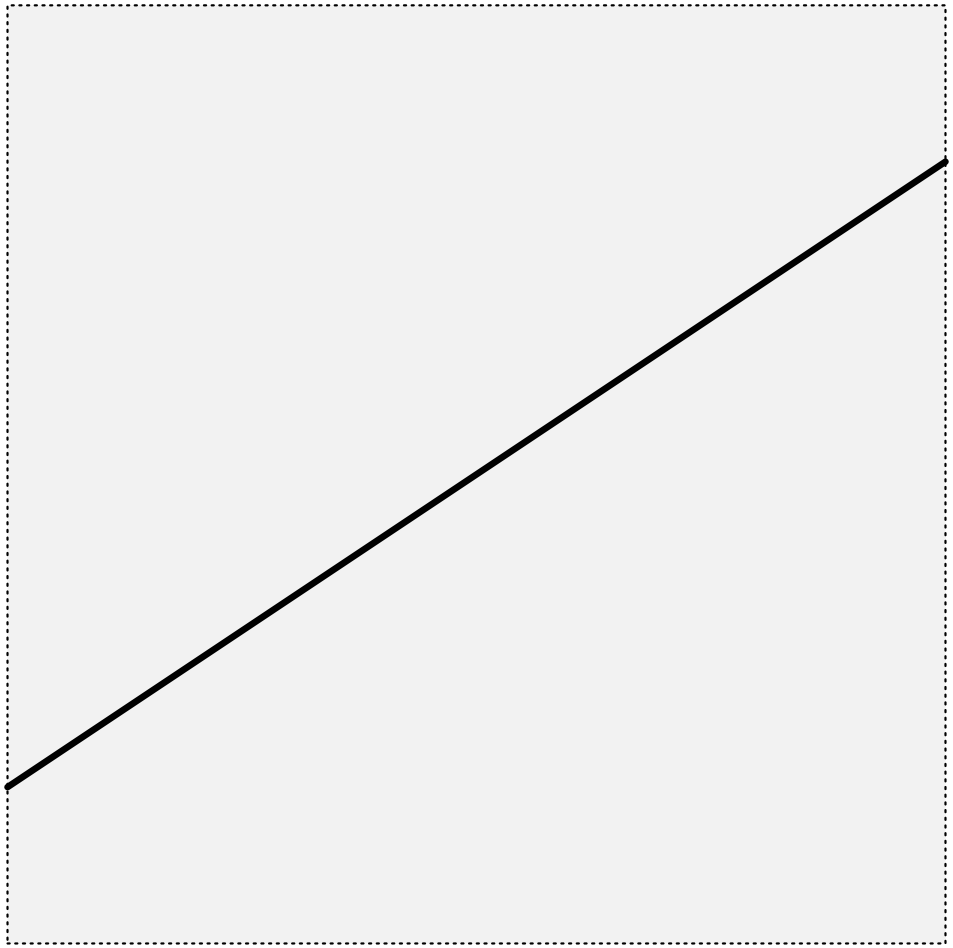
Poincaré half-plane model

Drawing the hyperbolic plane

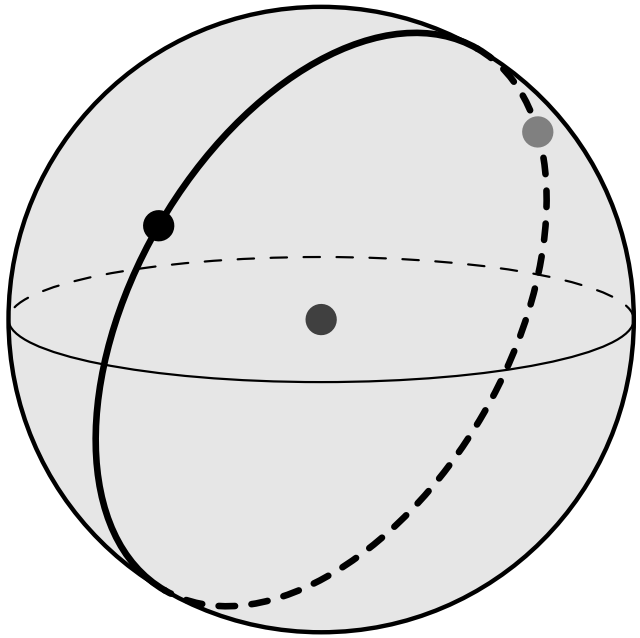
Geodesics

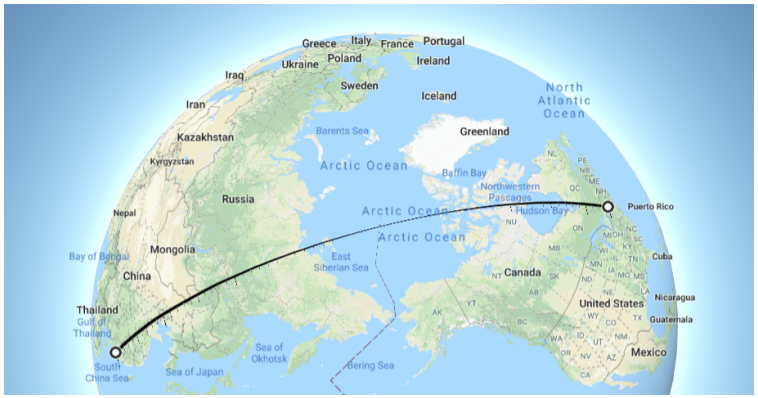
- “Straight lines” on a surface
- Well-defined on surfaces with metric
  - Shortest curve between two points (locally)
  - Tangent vectors are parallel
- Geodesics on surfaces with constant curvature?

Geodesics on the plane

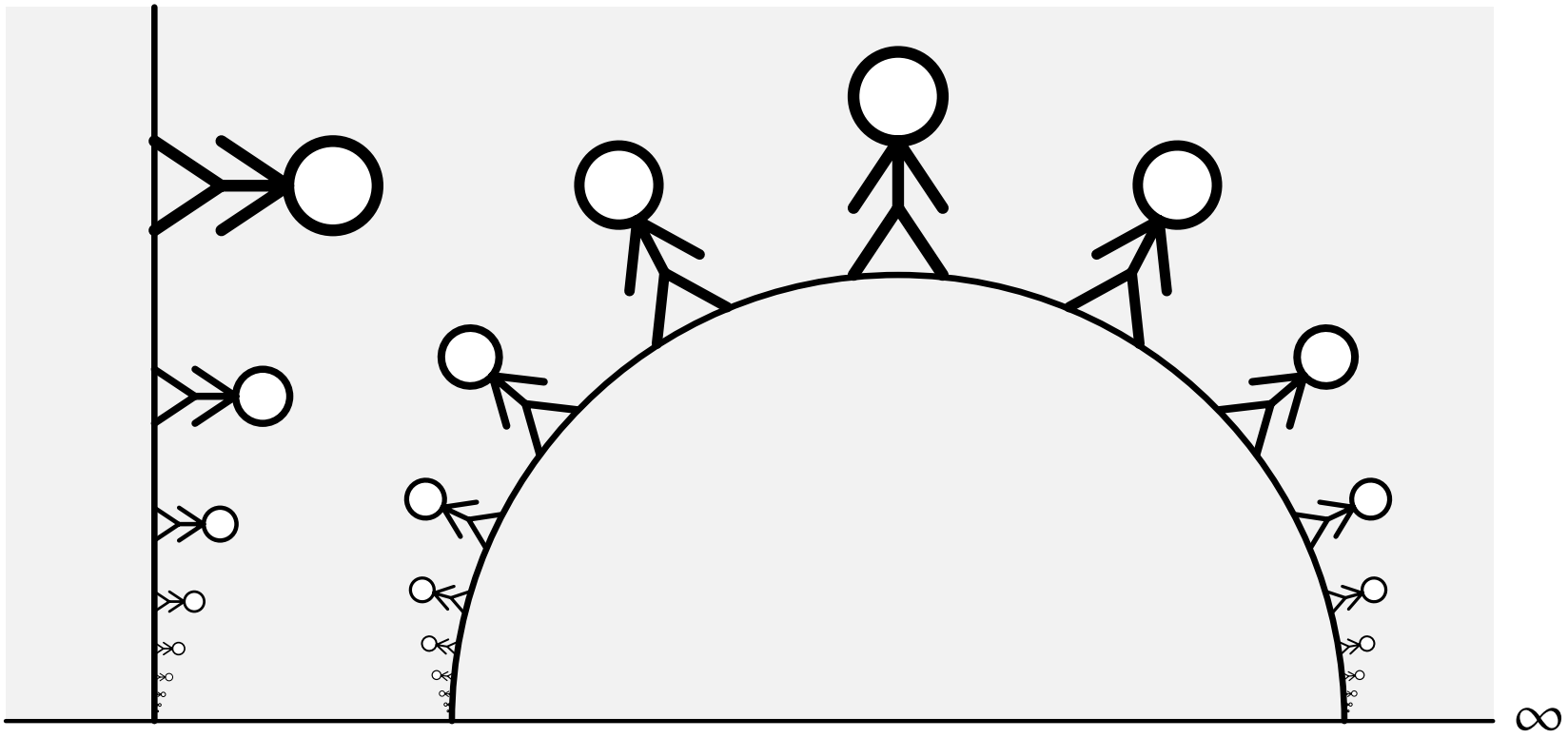


Geodesics on the sphere





Geodesics on the hyperbolic plane



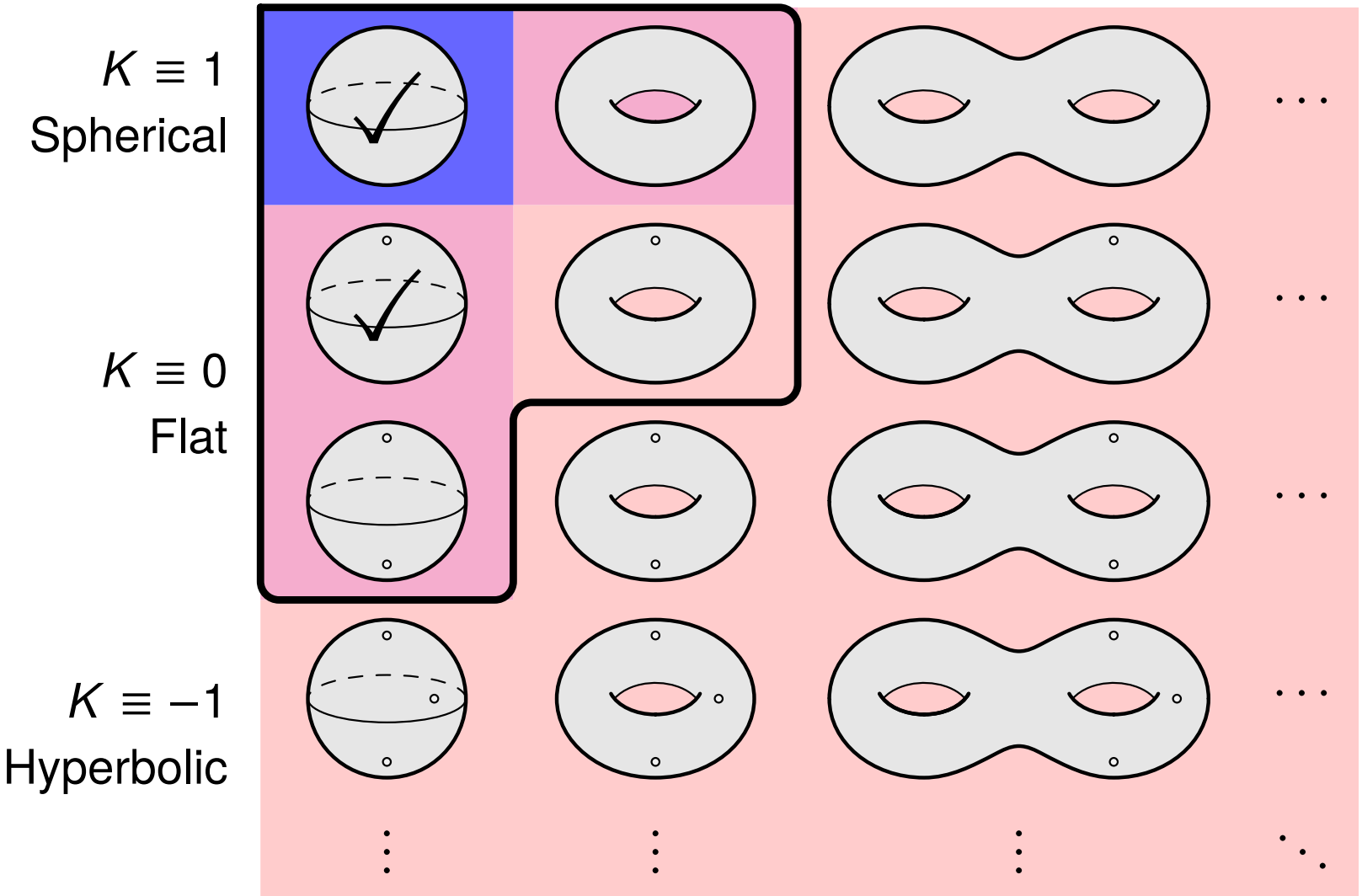
Geodesics: recap

Simple: does not self-intersect

Closed: loops periodically

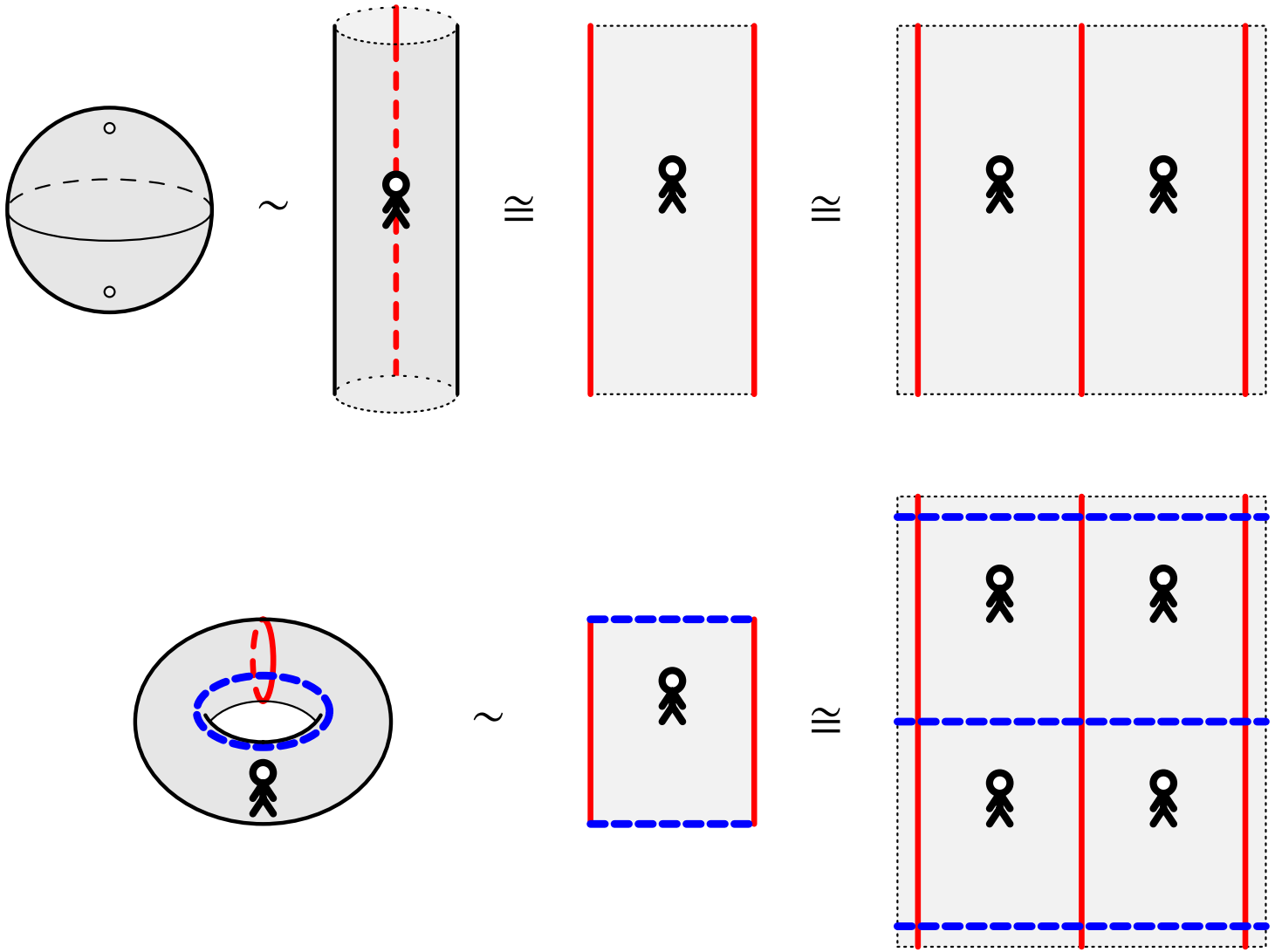
	Simple		Non-simple		
	Closed	Non-closed	Closed	Non-closed	Dense
Sphere	✓				
Euclidean plane		✓			
Hyperbolic plane		✓			

Surfaces of finite type

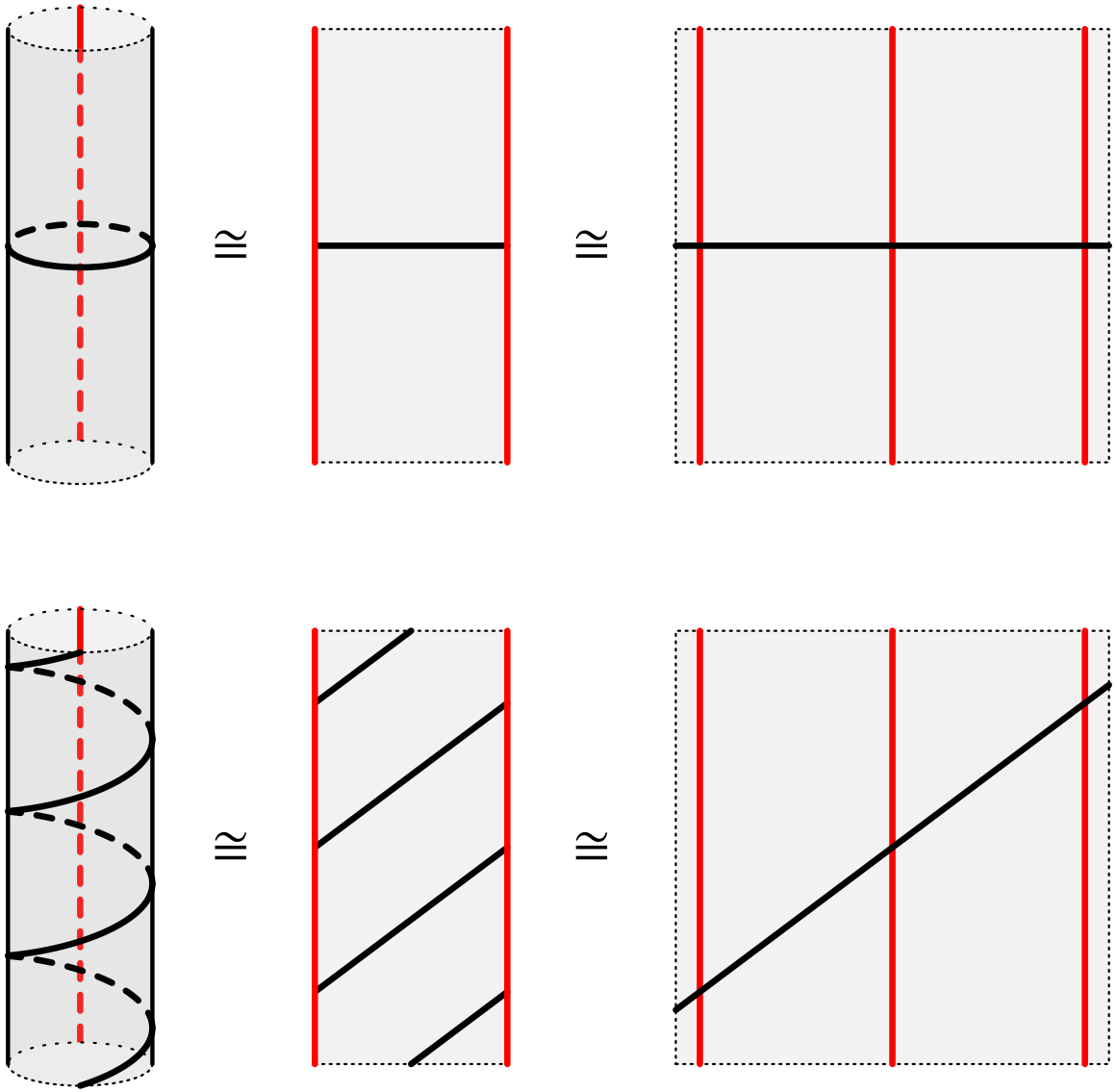


Theorem: Each surface above has a metric of constant curvature

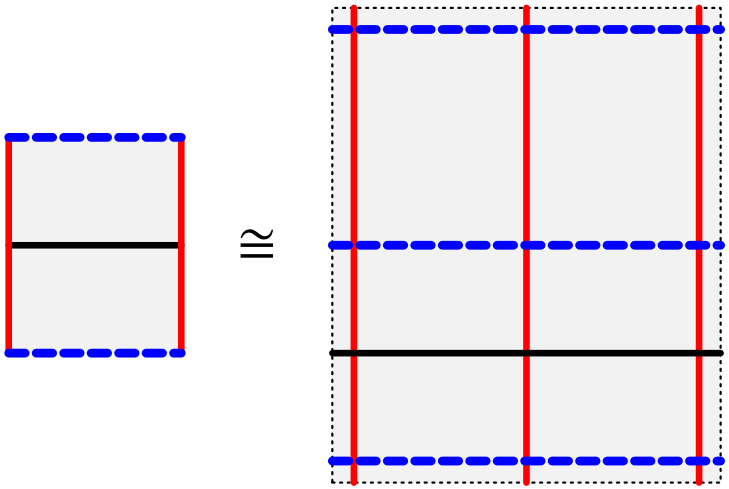
Flat surfaces

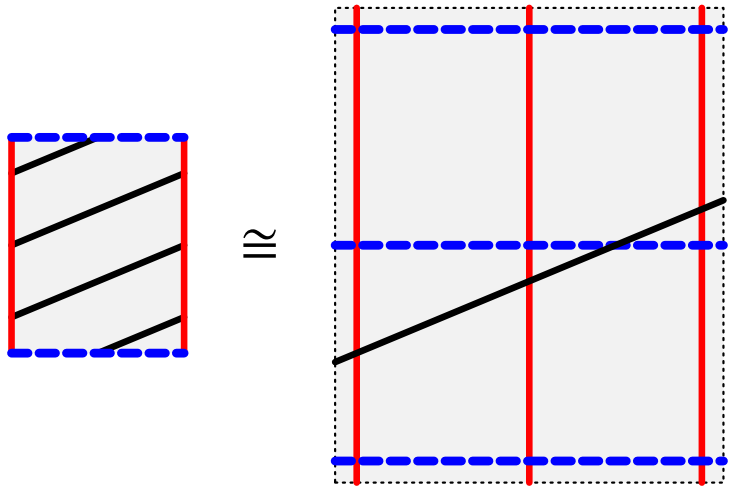


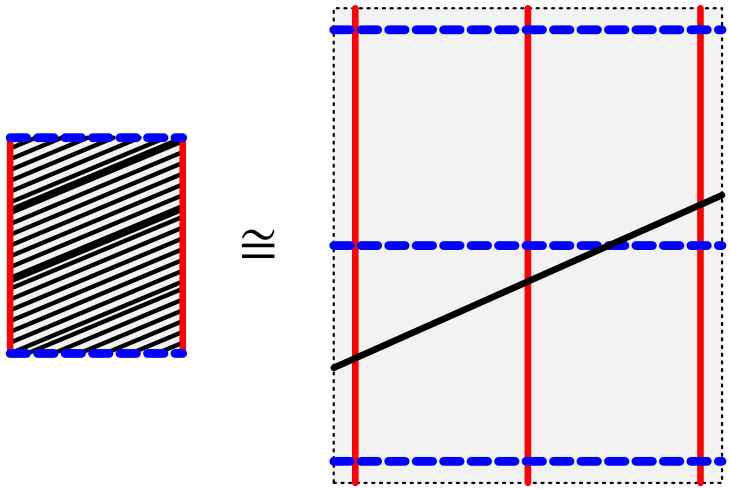
Geodesics on the flat cylinder



Geodesics on flat tori







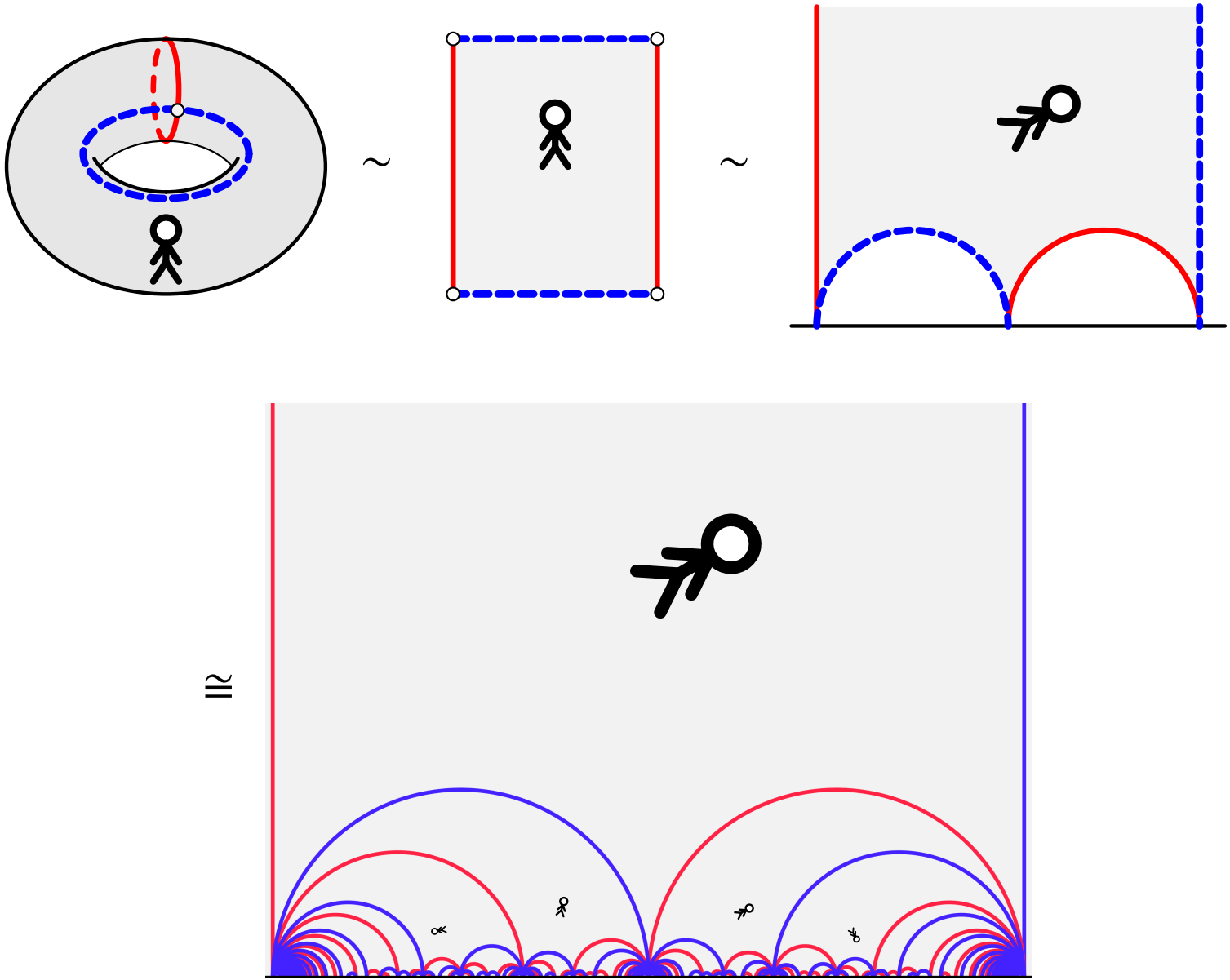
Geodesics: recap

Simple: does not self-intersect

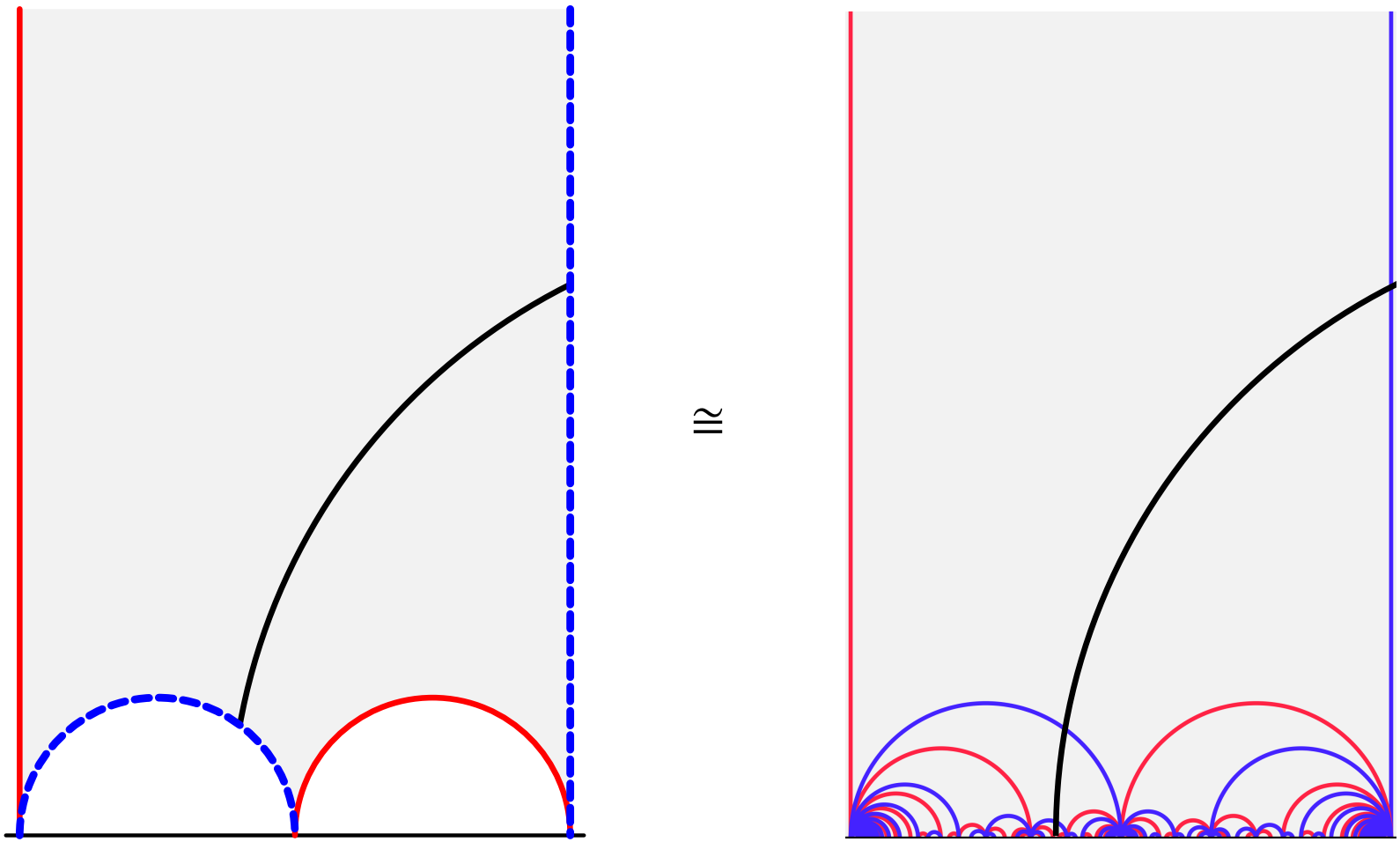
Closed: loops periodically

	Simple		Non-simple		
	Closed	Non-closed	Closed	Non-closed	Dense
Sphere	✓				
Euclidean plane		✓			
Flat cylinder	✓	✓			
Flat tori	✓	✓			✓
Hyperbolic plane		✓			
Hyperbolic once-punctured tori	✓	✓	✓	✓	✓

Hyperbolic once-punctured tori



Geodesics on hyperbolic once-punctured tori



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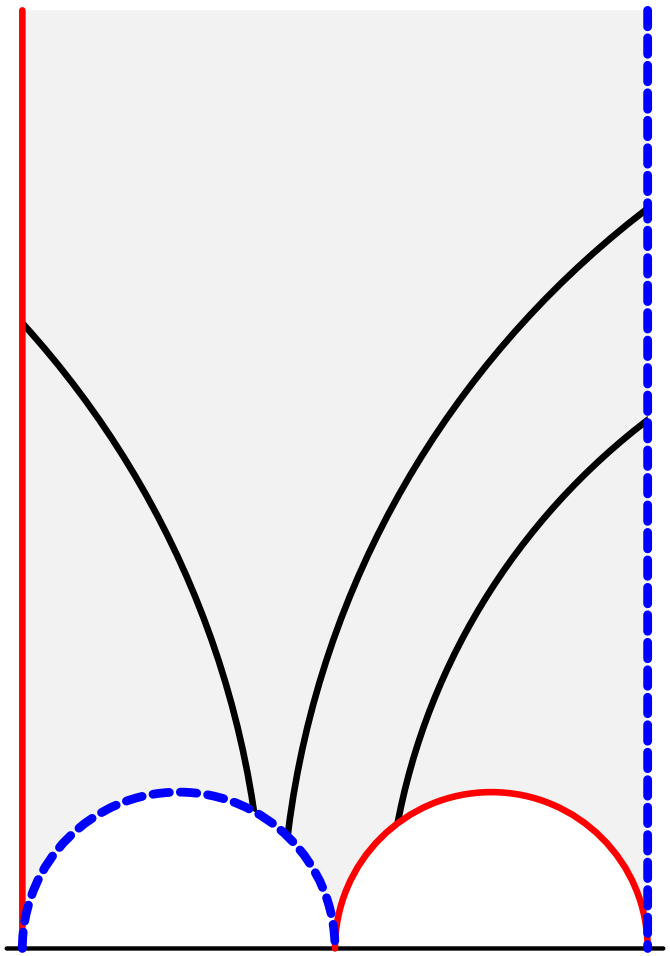

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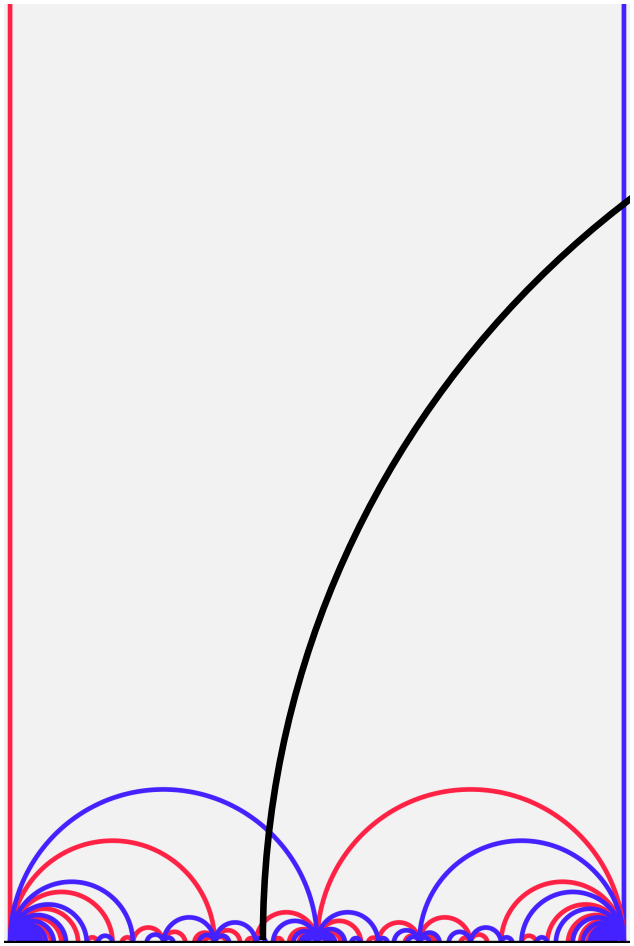
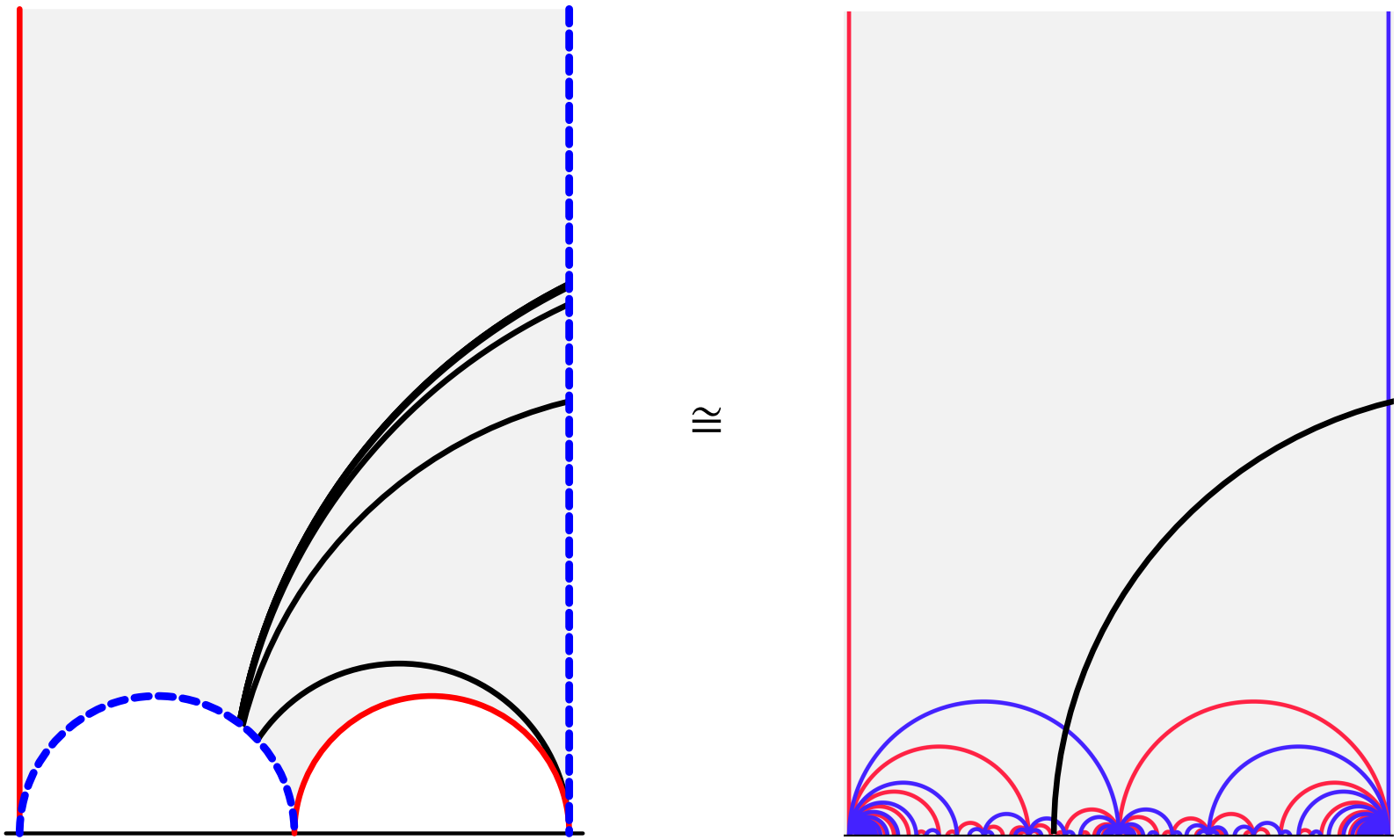
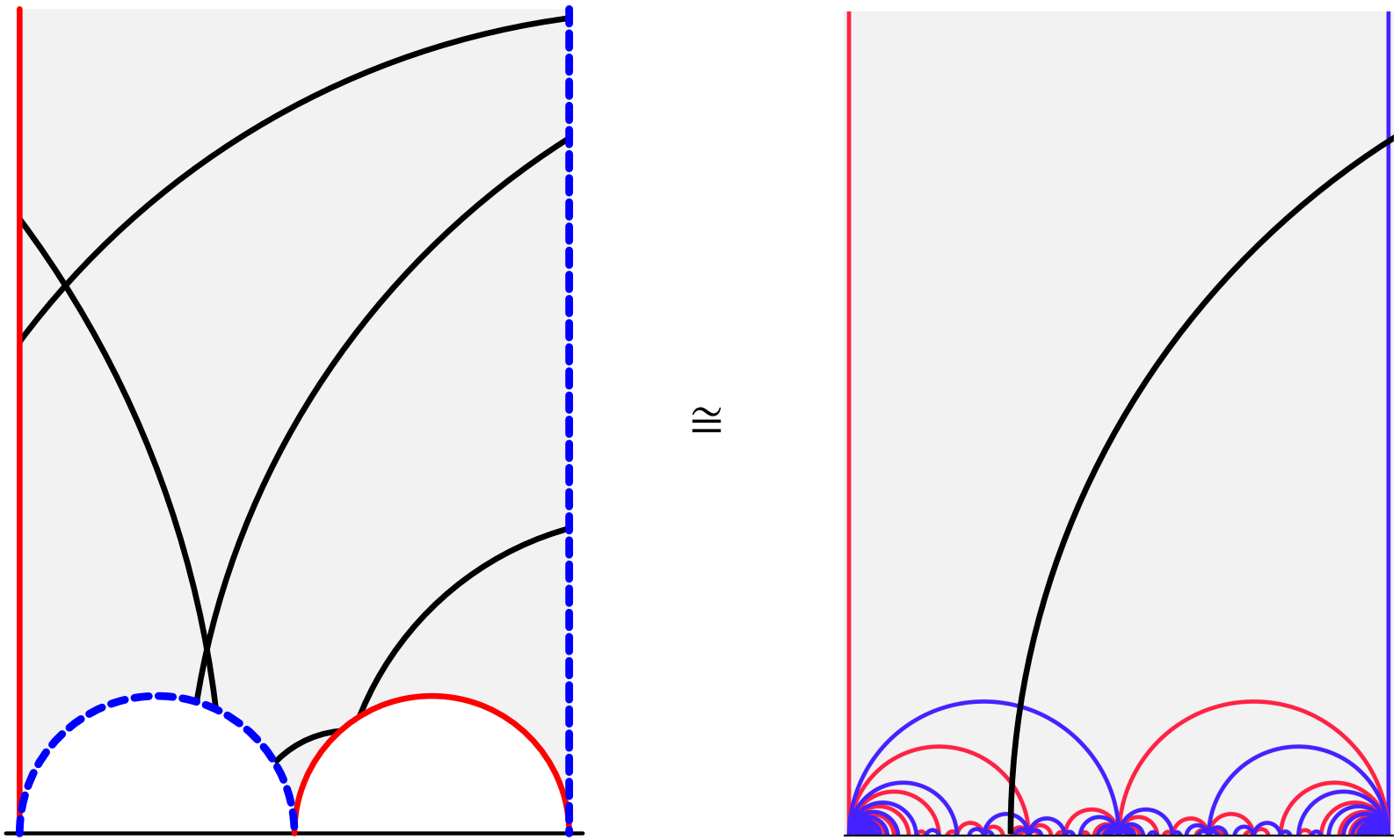


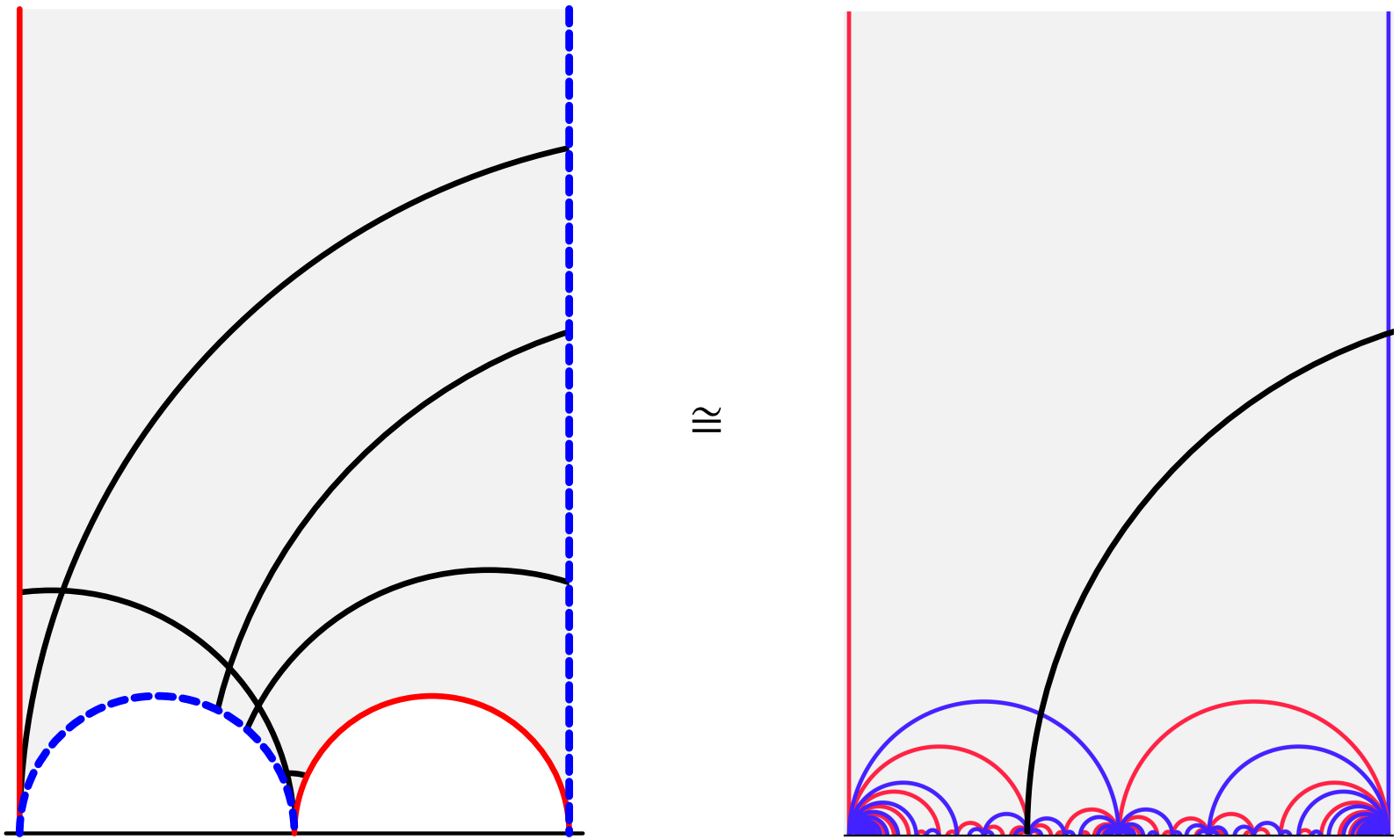
Geodesics on hyperbolic once-punctured tori



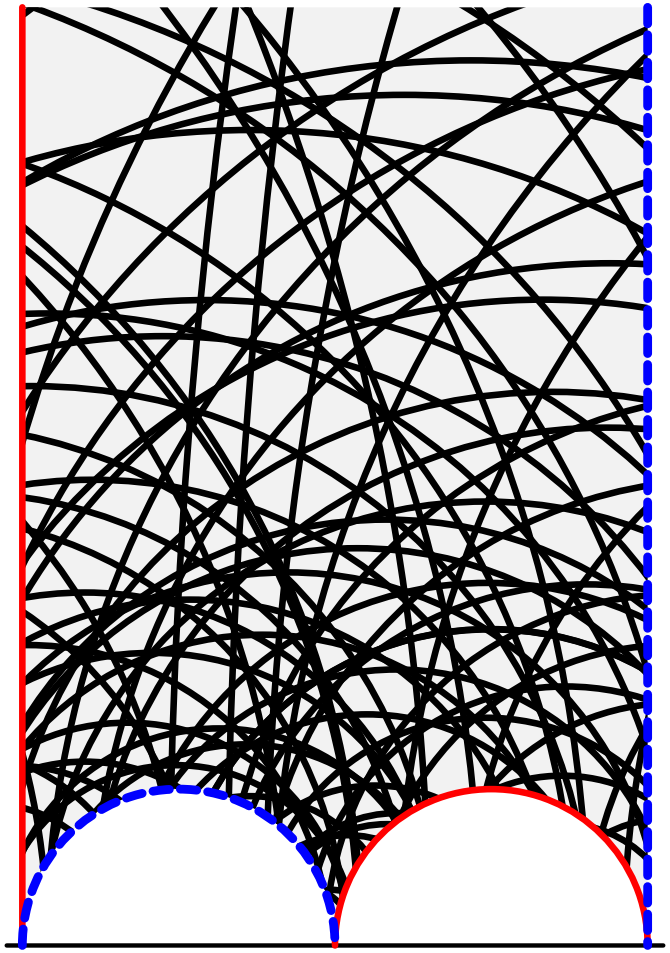
Geodesics on hyperbolic once-punctured tori



Geodesics on hyperbolic once-punctured tori



Geodesics on hyperbolic once-punctured tori



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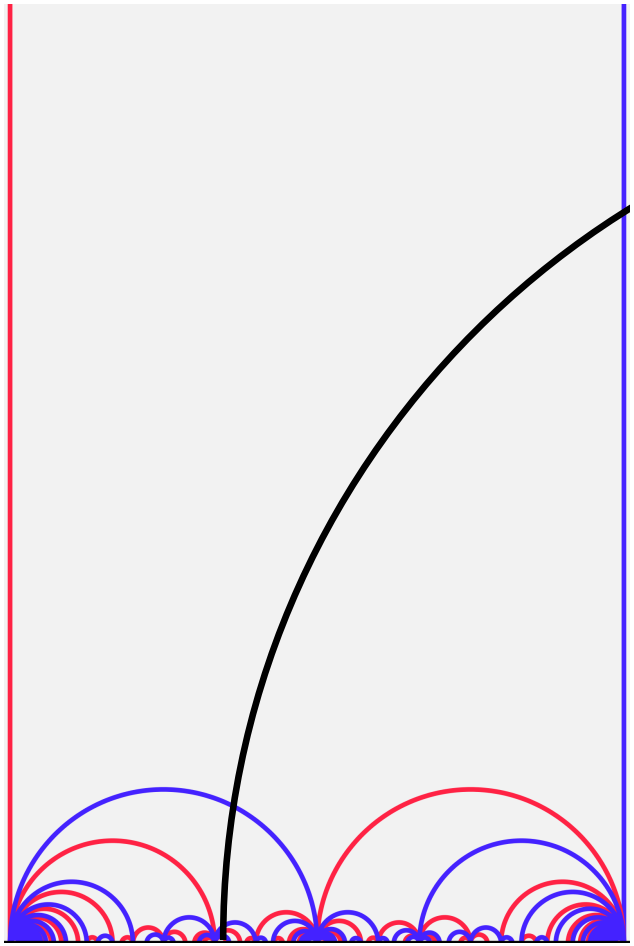

Questions on hyperbolic once-punctured tori

- For (simple) closed geodesics:
  - Structure/parametrisation
  - Enumeration (of length $\leq L$)
  - Relations between lengths
  - Connections to number theory
- Other types of geodesics:
  - Closed + almost simple
  - Both ends up the cusp
- Other hyperbolic surfaces
  - Surfaces of finite type
  - Surfaces with cone points

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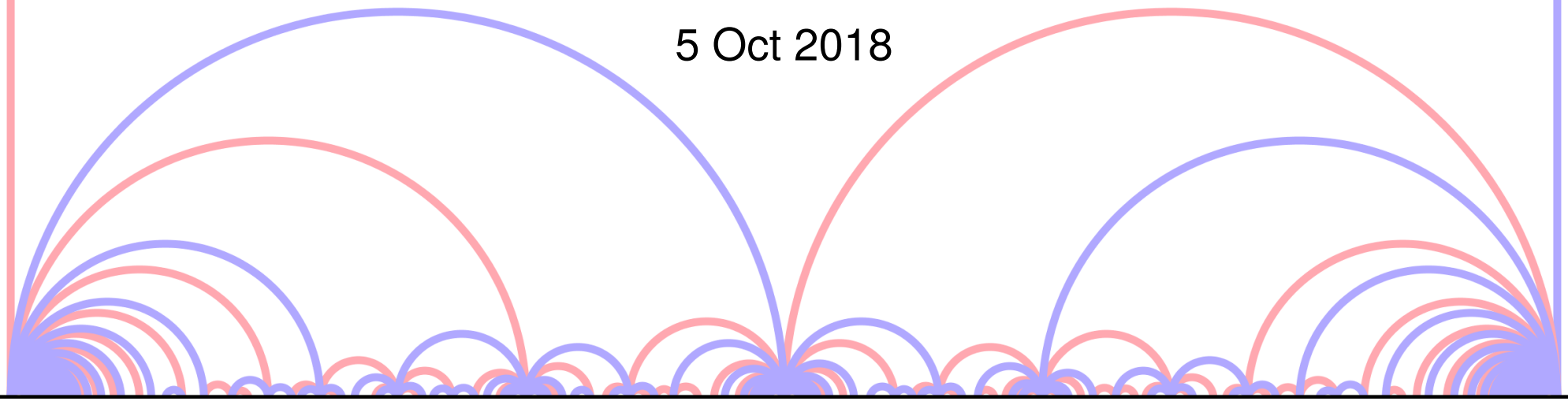
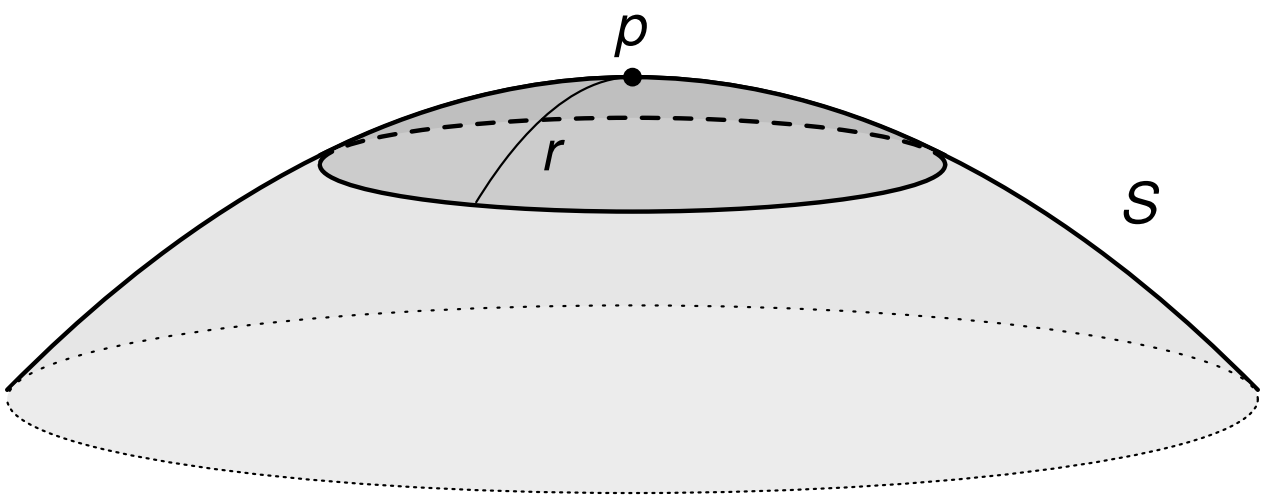


Curvature

Measures the area excess/deficit at a point



$$\text{Area} = \pi r^2 - K\frac{\pi r^4}{12} + \cdots \qquad K = \textbf{Gaussian curvature}$$

$K > 0$: Area deficit

$K < 0$: Area excess