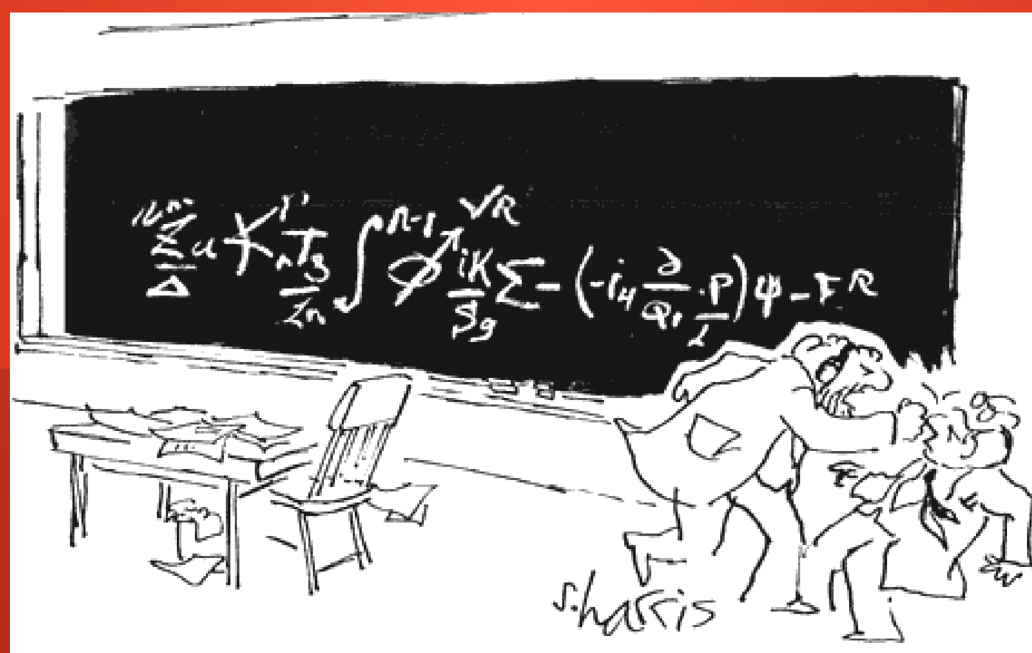
Why We Believe Mathematical Proofs

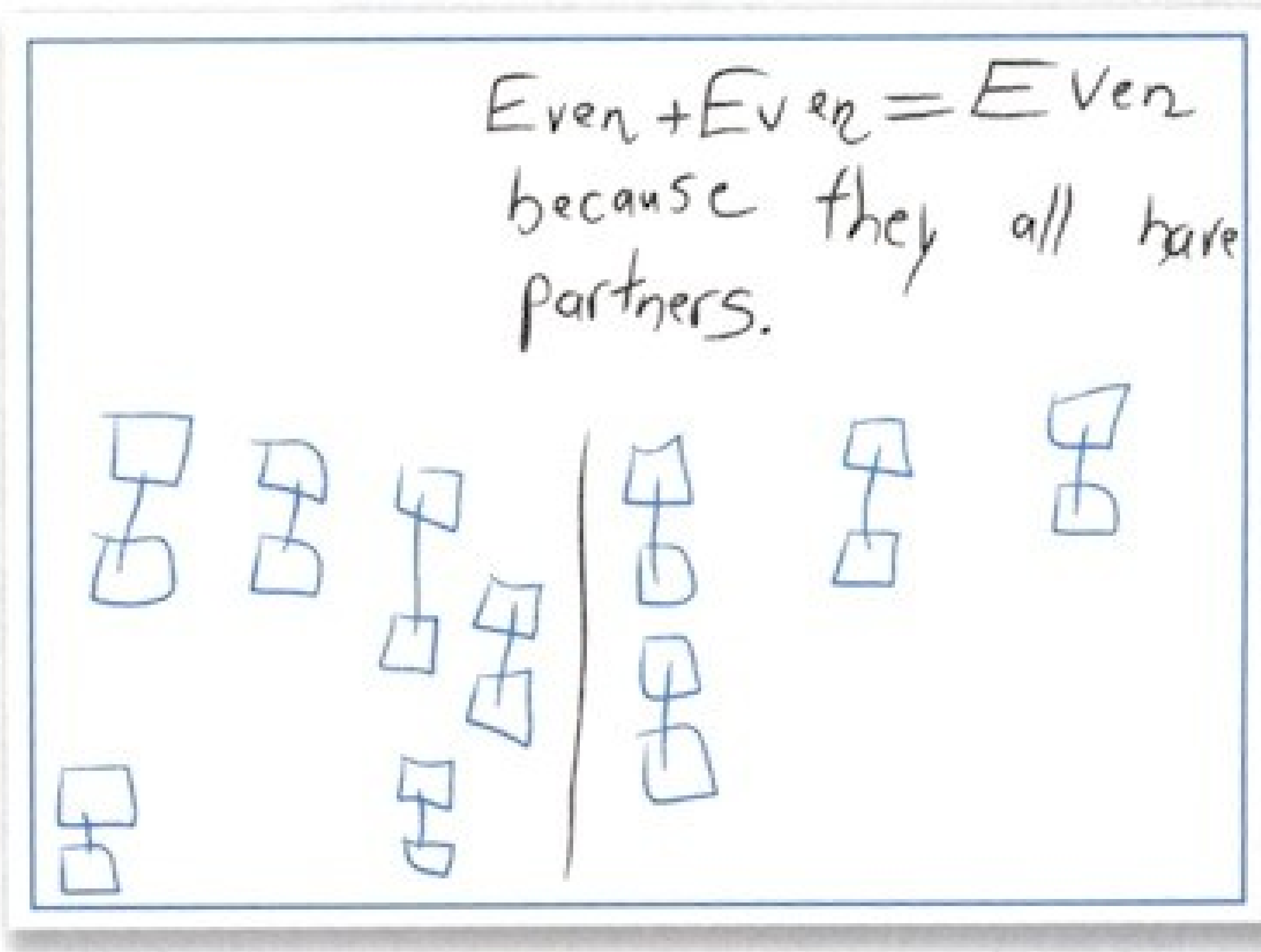
Ang Yan Sheng



"You want proof? I'll give you proof!"

Theorem: (YS Ang, 1998)

Even + even = even.



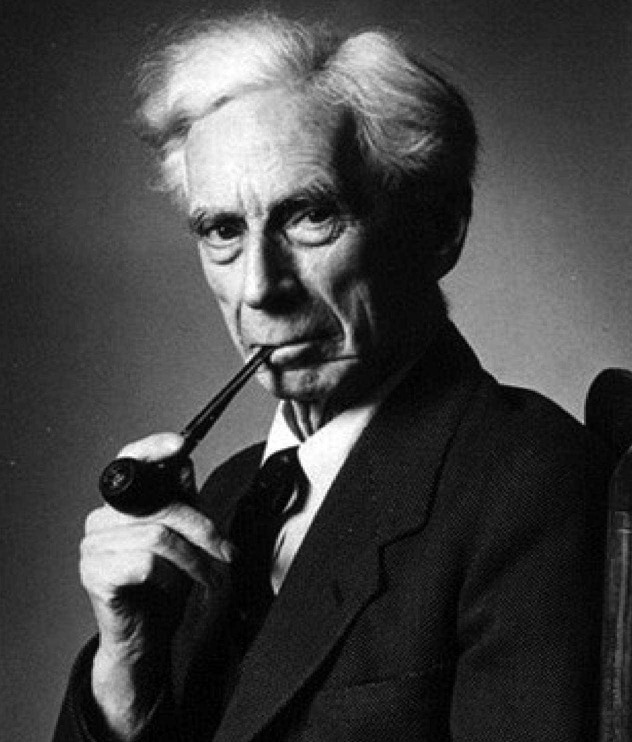
Sample Student Work

$$2a + 2b = 2(a + b)$$

Derivation (formal)

vs.

Intuition (informal)



***54·42.** $\vdash :: \alpha \in 2 . \supset :. \beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv . \beta \in \iota``\alpha$

Dem.

$\vdash . *54 \cdot 4 . \supset \vdash :: \alpha = \iota`x \cup \iota`y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \equiv : \beta = \Lambda . \vee . \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha : \exists ! \beta :$

[*24·53·56.*51·161] $\equiv : \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha$ (1)

$\vdash . *54 \cdot 25 . \text{Transp} . *52 \cdot 22 . \supset \vdash : x \neq y . \supset . \iota`x \cup \iota`y \neq \iota`x . \iota`x \cup \iota`y \neq \iota`y :$

[*13·12] $\supset \vdash : \alpha = \iota`x \cup \iota`y . x \neq y . \supset . \alpha \neq \iota`x . \alpha \neq \iota`y$ (2)

$\vdash . (1) . (2) . \supset \vdash :: \alpha = \iota`x \cup \iota`y . x \neq y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv : \beta = \iota`x . \vee . \beta = \iota`y :$

[*51·235] $\equiv : (\exists z) . z \in \alpha . \beta = \iota`z :$

[*37·6] $\equiv : \beta \in \iota``\alpha$ (3)

$\vdash . (3) . *11 \cdot 11 \cdot 35 . *54 \cdot 101 . \supset \vdash . \text{Prop}$

***54·43.** $\vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54 \cdot 26 . \supset \vdash :. \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

[*51·231] $\equiv . \iota`x \cap \iota`y = \Lambda .$

[*13·12] $\equiv . \alpha \cap \beta = \Lambda$ (1)

$\vdash . (1) . *11 \cdot 11 \cdot 35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda$ (2)

$\vdash . (2) . *11 \cdot 54 . *52 \cdot 1 . \supset \vdash . \text{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

Principia Mathematica (1910)

Assertion

Ref	Expression
2p2e4	$\vdash (2 + 2) = 4$

Proof of Theorem 2p2e4

Step	Hyp	Ref	Expression
1		df-2 8943	..3 $\vdash 2 = (1 + 1)$
2	1	oveq2i 5389	.2 $\vdash (2 + 2) = (2 + (1 + 1))$
3		df-4 8945	..3 $\vdash 4 = (3 + 1)$
4		df-3 8944	...4 $\vdash 3 = (2 + 1)$
5	4	oveq1i 5388	..3 $\vdash (3 + 1) = ((2 + 1) + 1)$
6		2cn 8953	...4 $\vdash 2 \in \mathbb{C}$
7		ax-1cn 8227	...4 $\vdash 1 \in \mathbb{C}$
8	6, 7, 7	addassi 8269	..3 $\vdash ((2 + 1) + 1) = (2 + (1 + 1))$
9	3, 5, 8	3eqtri 2106	.2 $\vdash 4 = (2 + (1 + 1))$
10	2, 9	eqtr4i 2105	1 $\vdash (2 + 2) = 4$

Metamath
25,933 steps

Theorem **opreq2i** 2368

Description: Equality inference for operations.

Hypothesis

Ref	Expression
opreq1i.1	$\vdash A = B$

Assertion

Ref	Expression
opreq2i	$\vdash (C F A) = (C F B)$

$A = B$

$2 = (1 + 1)$

2

$(C F A) = (C F B)$

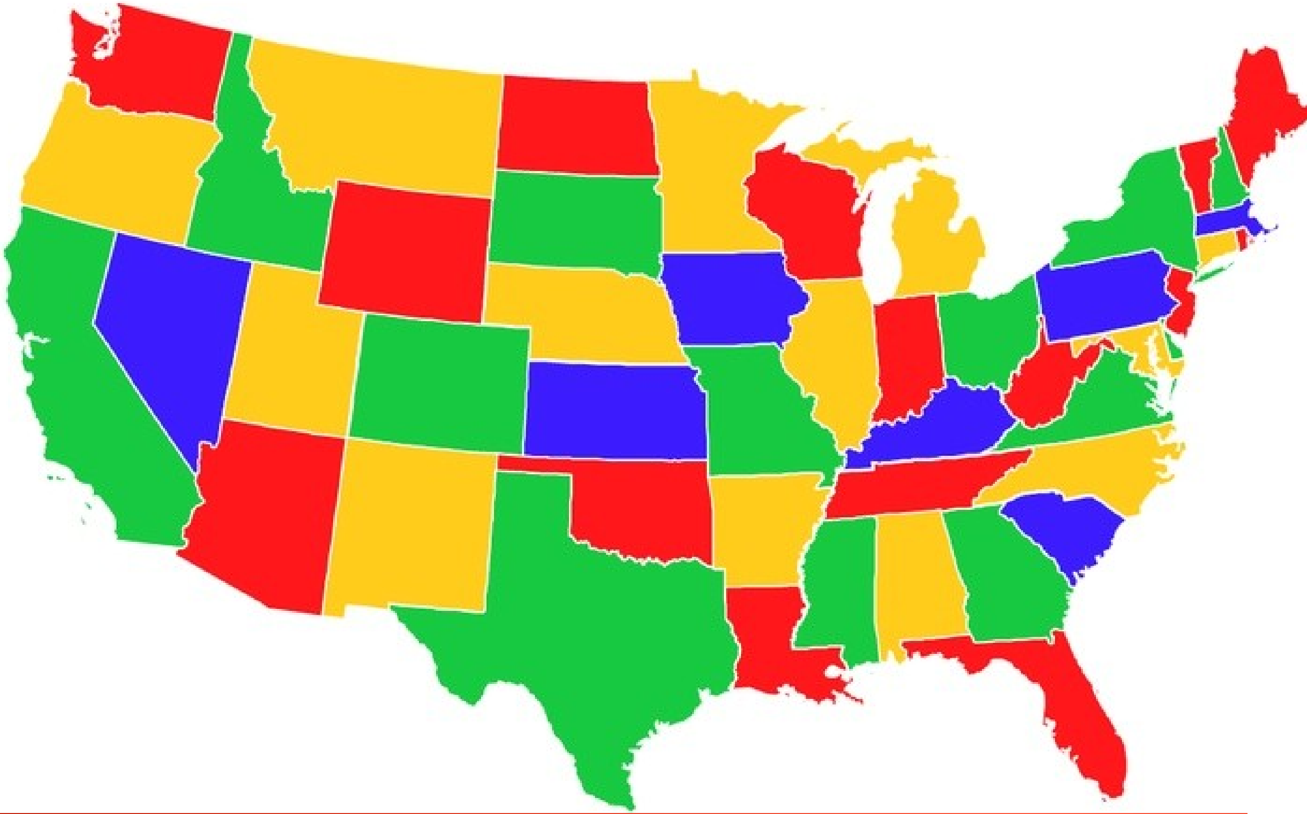
$(2 + 2) = (2 + (1 + 1))$

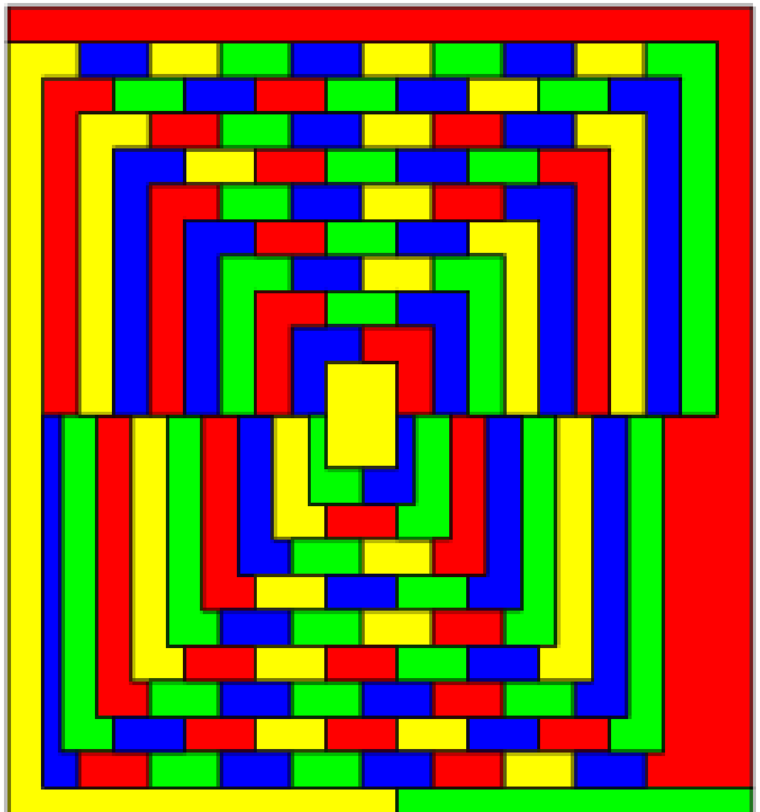
3

Proof of Theorem **2p2e4**

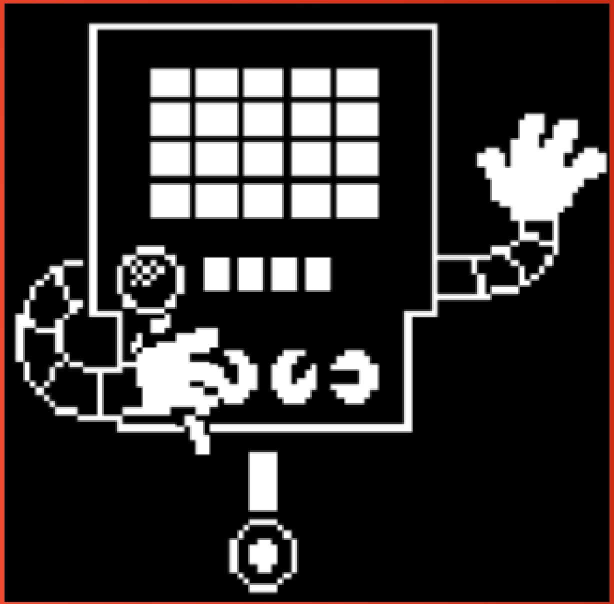
Step	Hyp	Ref	Expression
1		df-2 3348	$\vdash 2 = (1 + 1)$
2	1	opreq2i 2368	$\vdash (2 + 2) = (2 + (1 + 1))$
3		df-4 3350	$\vdash 4 = (3 + 1$ [illegible]

1





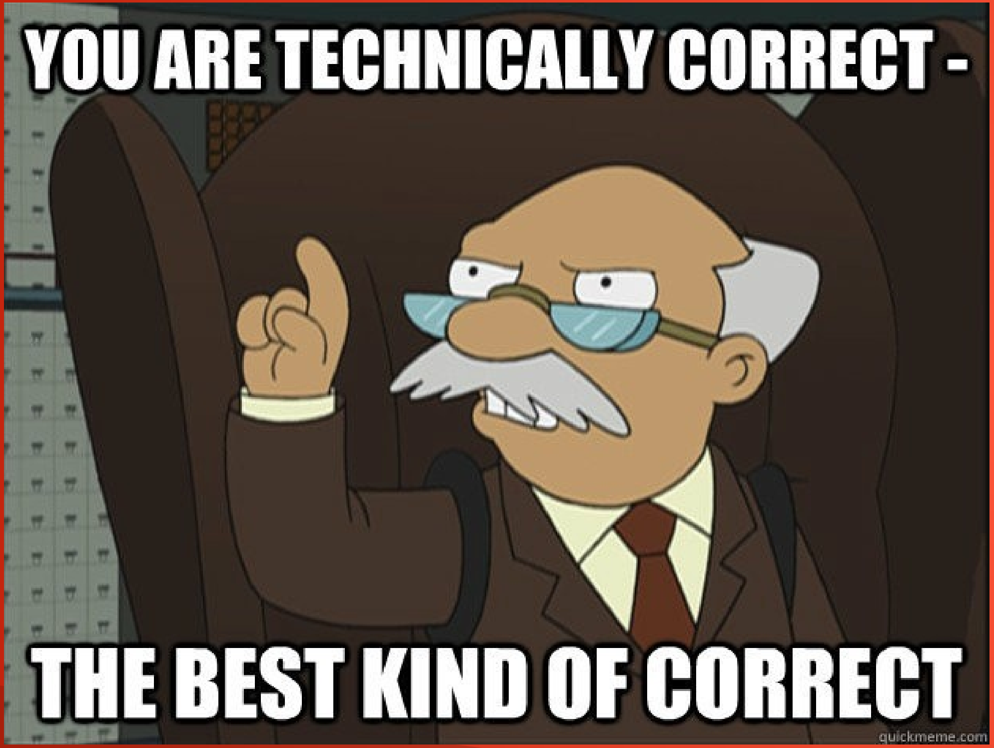
Appel-Haken
(1976)

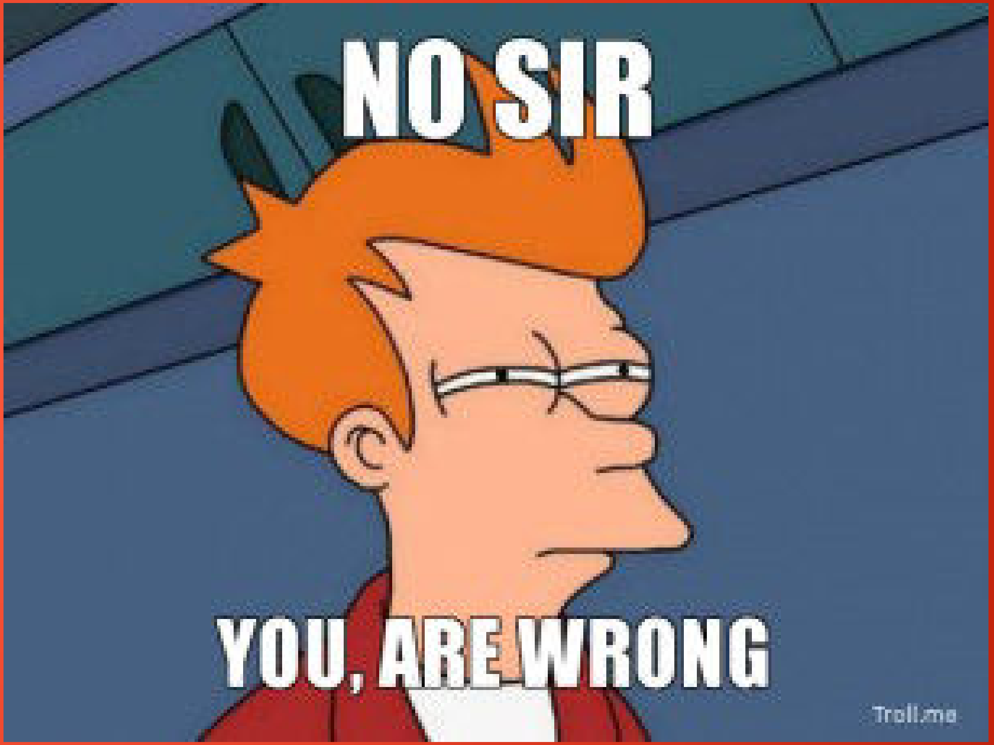


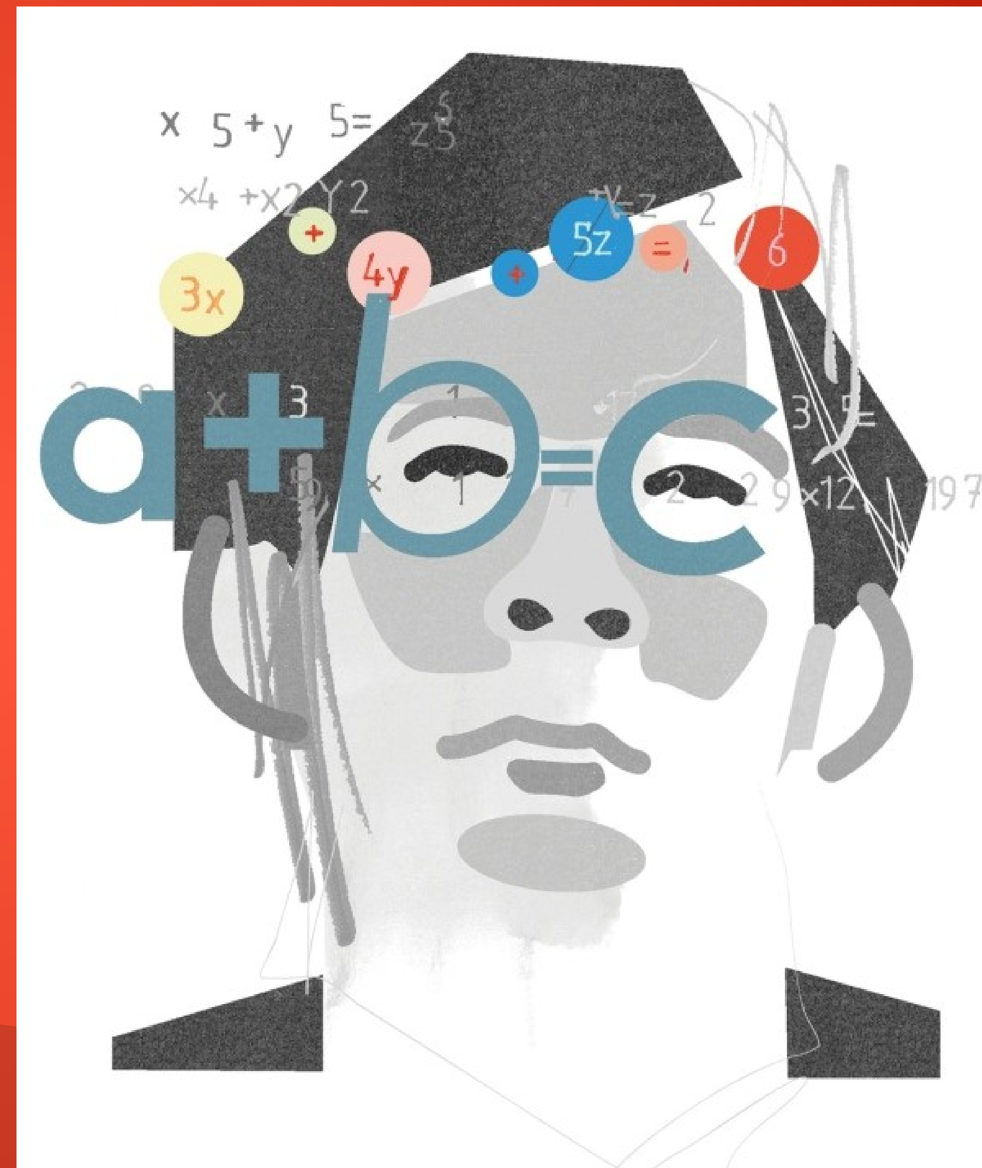
1,936 cases

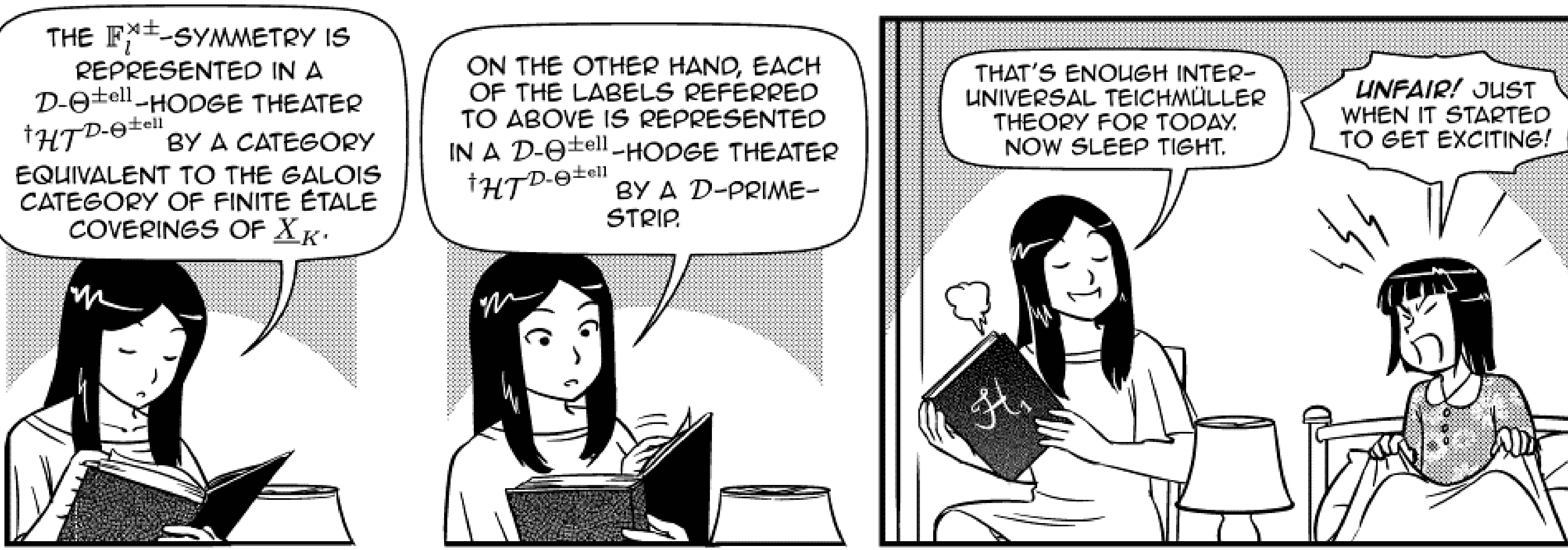
1,200 hours of computer time


WHAT?
WHY?
HOW?

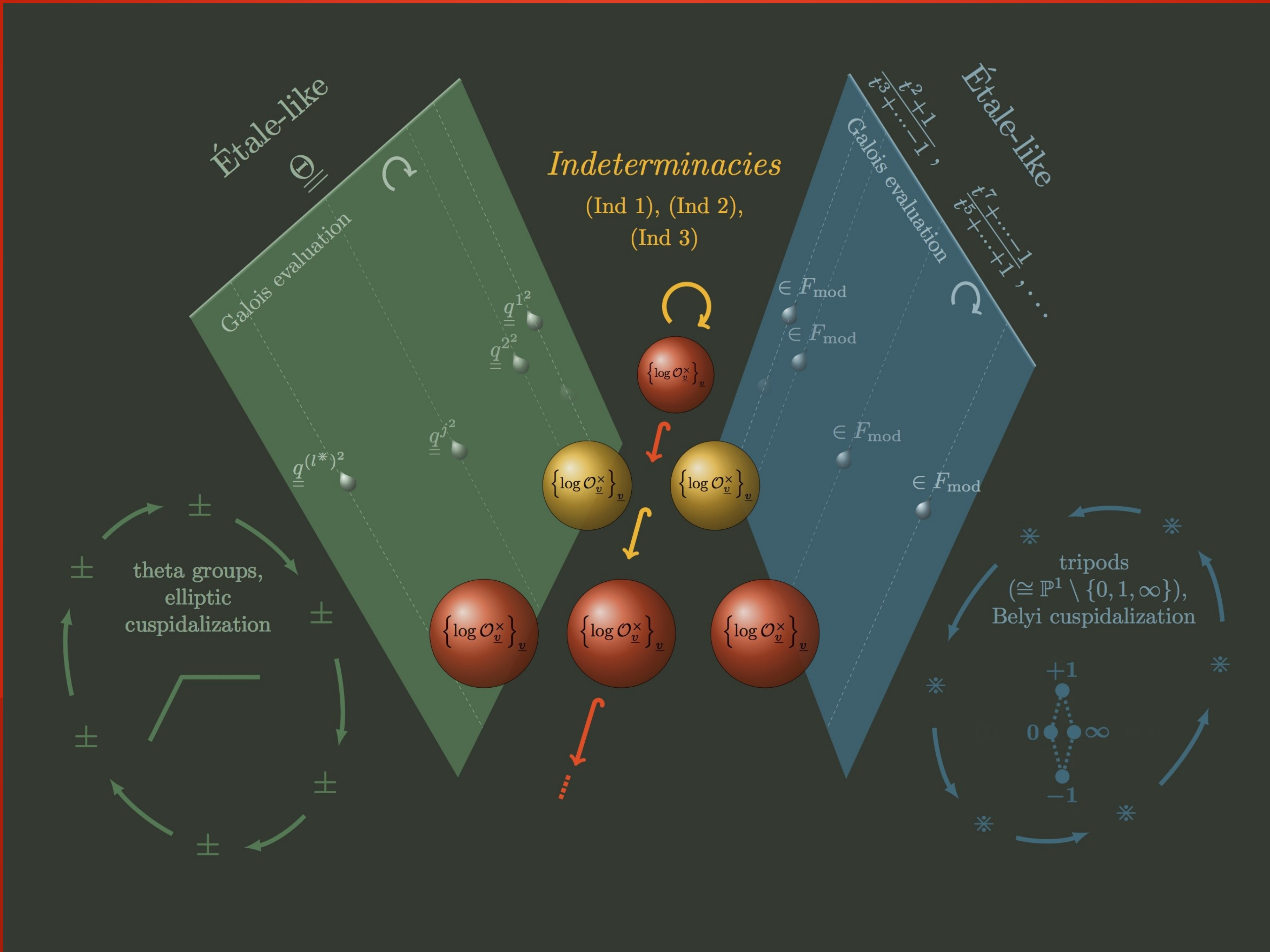

YOU ARE TECHNICALLY CORRECT -
THE BEST KIND OF CORRECT
quickmeme.com


NO SIR
YOU, ARE WRONG
Troll.me


x 5+y 5=
3x
4y
5z
6
a+b=c



Sandra and Woo by Oliver Knörzer (writer) and Powree (artist) – www.sandraandwoo.com


Étale-like
$\underline{\underline{\Theta}}$
Galois evaluation
$\underline{\underline{q}}^{1^2}$
$\underline{\underline{q}}^{2^2}$
$\underline{\underline{q}}^{j^2}$
$\underline{\underline{q}}^{(l^*)^2}$
Indeterminacies
(Ind 1), (Ind 2),
(Ind 3)
$\{\log \mathcal{O}^{\times}_{\underline{v}}\}_{\underline{v}}$
theta groups,
elliptic
cuspidalization
±
Étale-like
$\frac{t^2+1}{t^3+\dots-1}$, $\frac{t^7+\dots-1}{t^5+\dots+1}$, ...
Galois evaluation
$\in F_{\mathrm{mod}}$
tripods
$(\cong \mathbb{P}^1 \setminus \{0,1,\infty\})$,
Belyi cuspidalization
+1
0
∞
−1

Inter-universal Geometer

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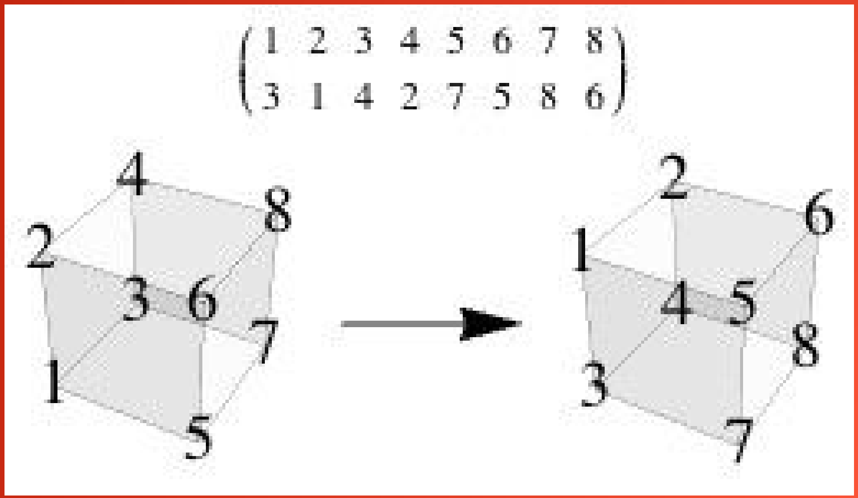
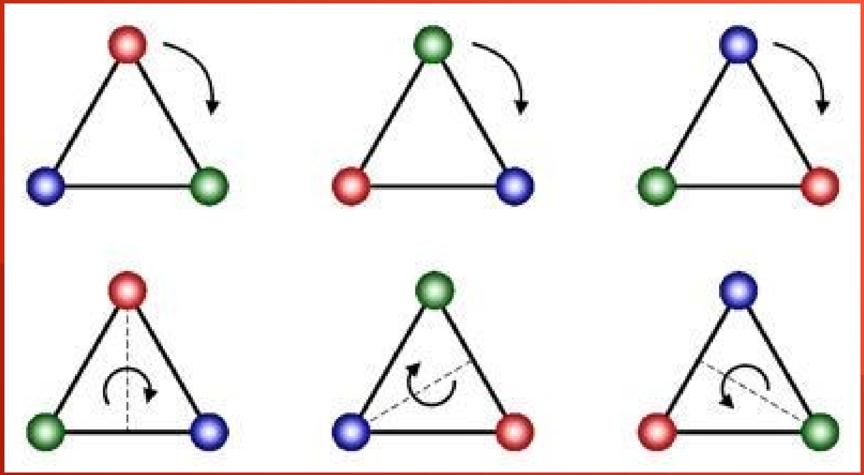
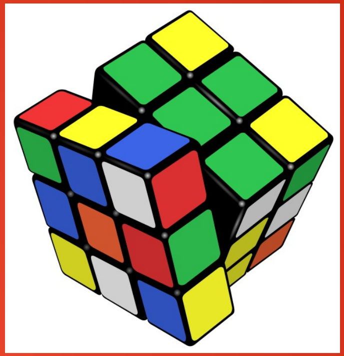
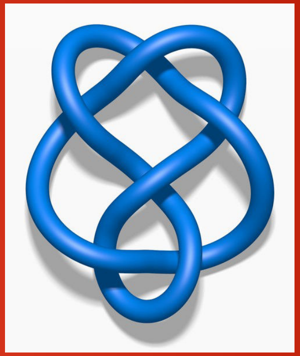
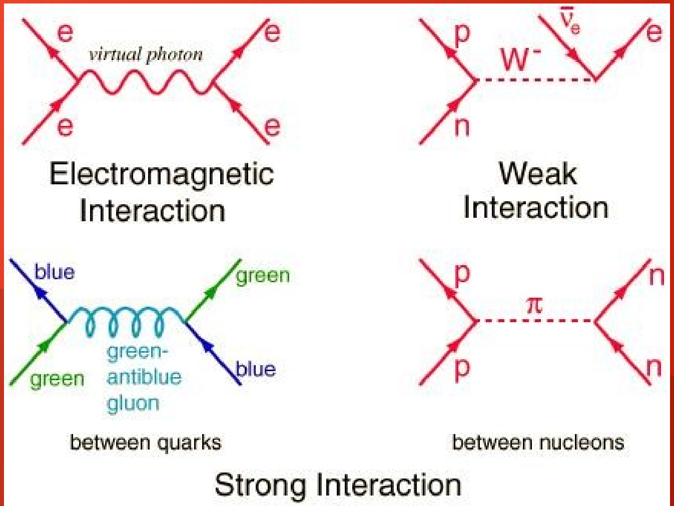
Shinichi Mochizuki

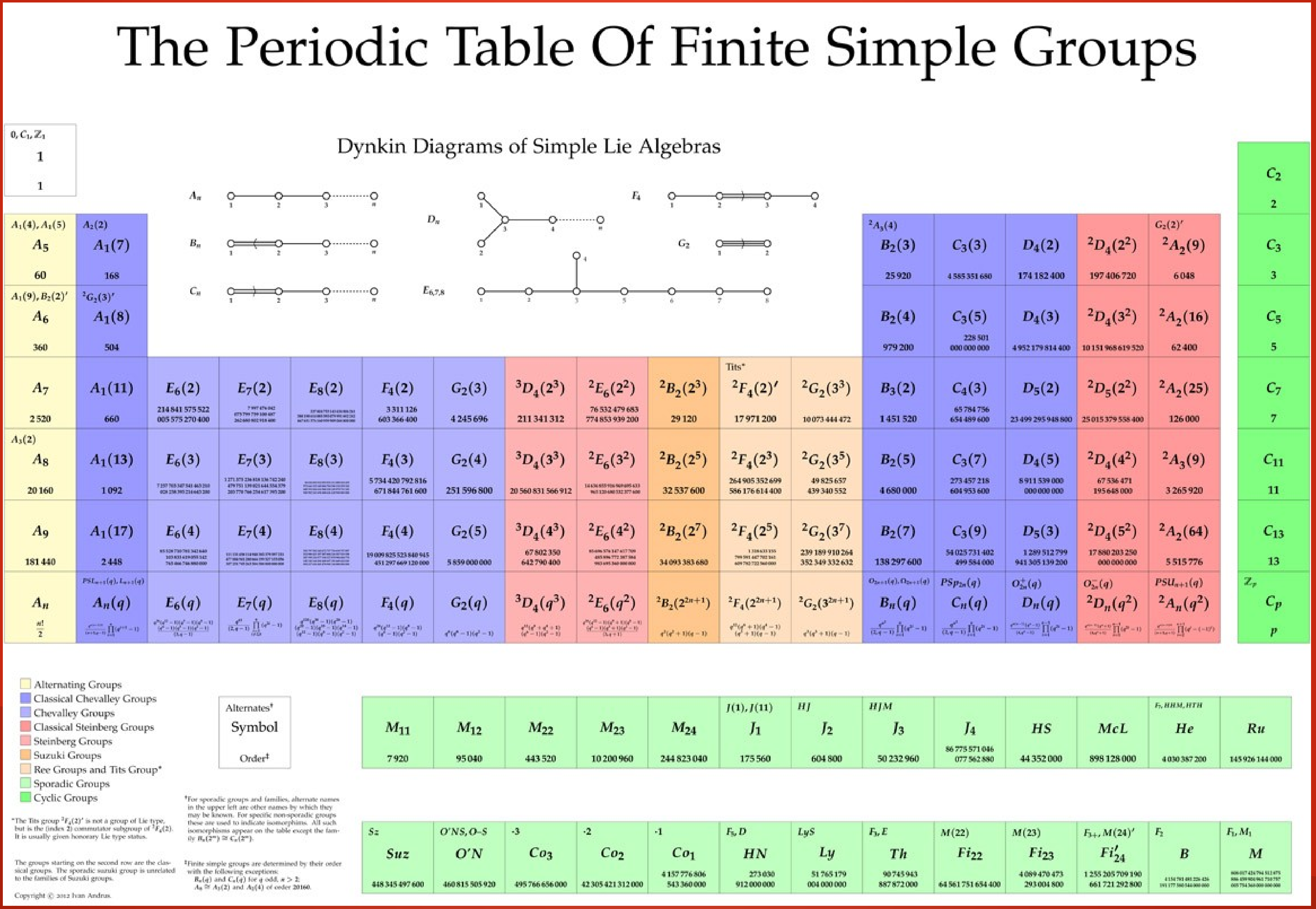

1 2 3 4 5 6 7 8
3 1 4 2 7 5 8 6








virtual photon
Electromagnetic Interaction
W⁻
Weak Interaction
blue
green
green-antiblue gluon
π
between quarks
between nucleons
Strong Interaction

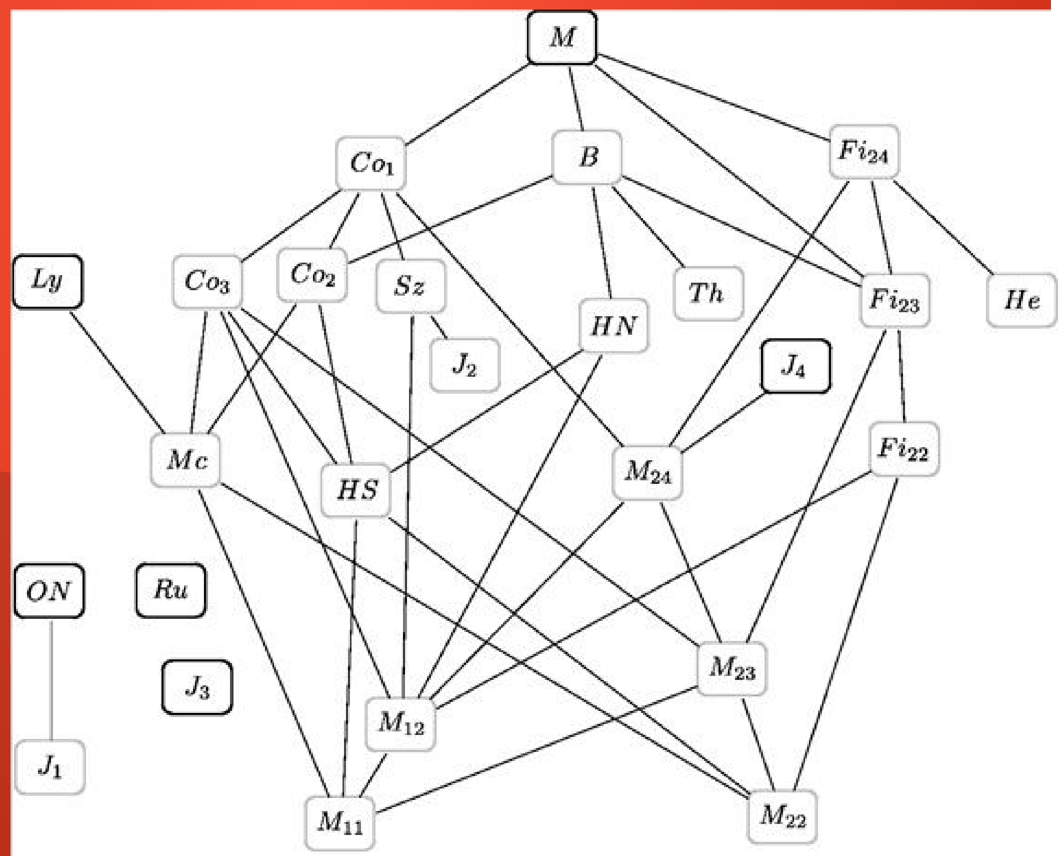

The Periodic Table Of Finite Simple Groups
Dynkin Diagrams of Simple Lie Algebras
A_n
B_n
C_n
D_n
$E_{6,7,8}$
F_4
G_2
$0, C_1, \mathbb{Z}_1$ 1 1
$A_1(4), A_1(5)$ A_5 60
$A_2(2)$ $A_1(7)$ 168
$A_1(9), B_2(2)'$ A_6 360
${}^2G_2(3)'$ $A_1(8)$ 504
A_7 2 520
$A_1(11)$ 660
$E_6(2)$ 214 841 575 522 005 575 270 400
$E_7(2)$
$E_8(2)$
$F_4(2)$ 3 311 126 603 366 400
$G_2(3)$ 4 245 696
${}^3D_4(2^3)$ 211 341 312
${}^2E_6(2^2)$ 76 532 479 683 774 853 939 200
${}^2B_2(2^3)$ 29 120
Tits* ${}^2F_4(2)'$ 17 971 200
${}^2G_2(3^3)$ 10 073 444 472
$A_3(2)$ A_8 20 160
$A_1(13)$ 1 092
$E_6(3)$
$E_7(3)$
$E_8(3)$
$F_4(3)$ 5 734 420 792 816 671 844 761 600
$G_2(4)$ 251 596 800
${}^3D_4(3^3)$ 20 560 831 566 912
${}^2E_6(3^2)$
${}^2B_2(2^5)$ 32 537 600
${}^2F_4(2^3)$ 264 905 352 699 586 176 614 400
${}^2G_2(3^5)$ 49 825 657 439 340 552
A_9 181 440
$A_1(17)$ 2 448
$E_6(4)$
$E_7(4)$
$E_8(4)$
$F_4(4)$ 19 009 825 523 840 945 451 297 669 120 000
$G_2(5)$ 5 859 000 000
${}^3D_4(4^3)$ 67 802 350 642 790 400
${}^2E_6(4^2)$
${}^2B_2(2^7)$ 34 093 383 680
${}^2F_4(2^5)$
${}^2G_2(3^7)$ 239 189 910 264 352 349 332 632
A_n $\frac{n!}{2}$
$PSL_{n+1}(q), L_{n+1}(q)$ $A_n(q)$
$E_6(q)$
$E_7(q)$
$E_8(q)$
$F_4(q)$
$G_2(q)$ $q^6(q^6-1)(q^2-1)$
${}^3D_4(q^3)$
${}^2E_6(q^2)$
${}^2B_2(2^{2n+1})$ $q^2(q^2+1)(q-1)$
${}^2F_4(2^{2n+1})$
${}^2G_2(3^{2n+1})$ $q^3(q^3+1)(q-1)$
${}^2A_3(4)$ $B_2(3)$ 25 920
$C_3(3)$ 4 585 351 680
$D_4(2)$ 174 182 400
${}^2D_4(2^2)$ 197 406 720
$G_2(2)'$ ${}^2A_2(9)$ 6 048
$B_2(4)$ 979 200
$C_3(5)$ 228 501 000 000 000
$D_4(3)$ 4 952 179 814 400
${}^2D_4(3^2)$ 10 151 968 619 520
${}^2A_2(16)$ 62 400
$B_3(2)$ 1 451 520
$C_4(3)$ 65 784 756 654 489 600
$D_5(2)$ 23 499 295 948 800
${}^2D_5(2^2)$ 25 015 379 558 400
${}^2A_2(25)$ 126 000
$B_2(5)$ 4 680 000
$C_3(7)$ 273 457 218 604 953 600
$D_4(5)$ 8 911 539 000 000 000 000
${}^2D_4(4^2)$ 67 536 471 195 648 000
${}^2A_3(9)$ 3 265 920
$B_2(7)$ 138 297 600
$C_3(9)$ 54 025 731 402 499 584 000
$D_5(3)$ 1 289 512 799 941 305 139 200
${}^2D_4(5^2)$ 17 880 203 250 000 000 000
${}^2A_2(64)$ 5 515 776
$O_{2n+1}(q), \Omega_{2n+1}(q)$ $B_n(q)$
$PSp_{2n}(q)$ $C_n(q)$
$O^+_{2n}(q)$ $D_n(q)$
$O^-_{2n}(q)$ ${}^2D_n(q^2)$
$PSU_{n+1}(q)$ ${}^2A_n(q^2)$
C_2 2
C_3 3
C_5 5
C_7 7
C_{11} 11
C_{13} 13
$\mathbb{Z}_p$ C_p p
Alternating Groups
Classical Chevalley Groups
Chevalley Groups
Classical Steinberg Groups
Steinberg Groups
Suzuki Groups
Ree Groups and Tits Group*
Sporadic Groups
Cyclic Groups
Alternates† Symbol Order‡
M_{11} 7 920
M_{12} 95 040
M_{22} 443 520
M_{23} 10 200 960
M_{24} 244 823 040
$J(1), J(11)$ J_1 175 560
HJ J_2 604 800
HJM J_3 50 232 960
J_4 86 775 571 046 077 562 880
HS 44 352 000
McL 898 128 000
F_7, HHM, HTH He 4 030 387 200
Ru 145 926 144 000
Sz Suz 448 345 497 600
$O'NS, O\text{–}S$ $O'N$ 460 815 505 920
$\cdot 3$ Co_3 495 766 656 000
$\cdot 2$ Co_2 42 305 421 312 000
$\cdot 1$ Co_1 4 157 776 806 543 360 000
F_5, D HN 273 030 912 000 000
LyS Ly 51 765 179 004 000 000
F_3, E Th 90 745 943 887 872 000
$M(22)$ Fi_{22} 64 561 751 654 400
$M(23)$ Fi_{23} 4 089 470 473 293 004 800
$F_{3+}, M(24)'$ Fi'_{24} 1 255 205 709 190 661 721 292 800
F_2 B
F_1, M_1 M
*The Tits group ${}^2F_4(2)'$ is not a group of Lie type, but is the (index 2) commutator subgroup of ${}^2F_4(2)$. It is usually given honorary Lie type status.
The groups starting on the second row are the classical groups. The sporadic suzuki group is unrelated to the families of Suzuki groups.
Copyright © 2012 Ivan Andrus.
†For sporadic groups and families, alternate names in the upper left are other names by which they may be known. For specific non-sporadic groups these are used to indicate isomorphisms. All such isomorphisms appear on the table except the family $B_n(2^m) \cong C_n(2^m)$.
‡Finite simple groups are determined by their order with the following exceptions: $B_n(q)$ and $C_n(q)$ for q odd, $n > 2$; $A_8 \cong A_3(2)$ and $A_2(4)$ of order 20160.

Enormous Theorem (1983)

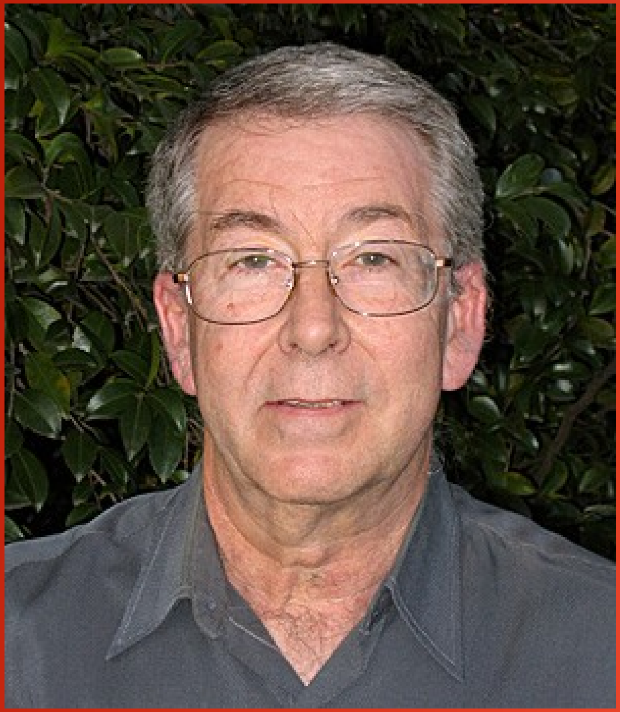
Every finite simple group belongs to either:

- one of 18 different infinite families (omitted); or
- the list of 26 exceptions.

10,000+ pages
500+ papers
100+ reseachers

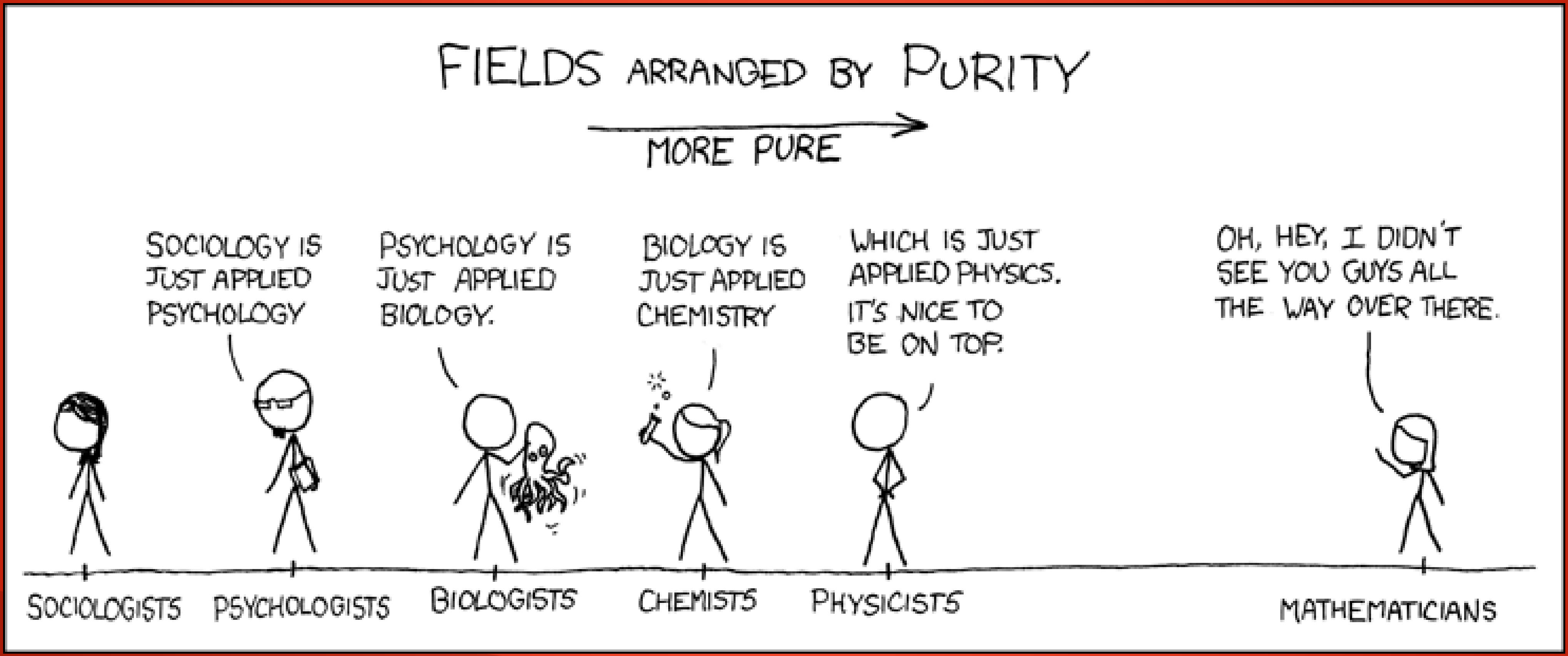


- Mathieu group M_{11}
- Mathieu group M_{12}
- Janko group J_1
- Mathieu group M_{22}
- Janko group $J_2 = HJ$
- Mathieu group M_{23}
- Higman-Sims group HS
- Janko group J_3
- Mathieu group M_{24}
- McLaughlin group McL
- Held group He
- Rudvalis Group Ru
- Suzuki group Suz
- O'Nan group O'N
- Conway group Co_3
- Conway group Co_2
- Fischer group Fi_{22}
- Harada-Norton group HN
- Lyons Group Ly
- Thompson Group Th
- Fischer group Fi_{23}
- Conway group Co_1
- Janko group J_4
- Fischer group Fi_{24}'
- baby monster group B
- monster group M



Aschbacher-Smith
(2004)

1,221 pages


FIELDS ARRANGED BY PURITY
MORE PURE
SOCIOLOGY IS JUST APPLIED PSYCHOLOGY
PSYCHOLOGY IS JUST APPLIED BIOLOGY.
BIOLOGY IS JUST APPLIED CHEMISTRY
WHICH IS JUST APPLIED PHYSICS. IT'S NICE TO BE ON TOP.
OH, HEY, I DIDN'T SEE YOU GUYS ALL THE WAY OVER THERE.
SOCIOLOGISTS
PSYCHOLOGISTS
BIOLOGISTS
CHEMISTS
PHYSICISTS
MATHEMATICIANS